

ANIMAL BREEDING NOTES

CHAPTER 17

APPROXIMATIONS TO THE ANIMAL MODEL

Data sets in animal breeding are frequently very large and sometimes parts of a data set cannot be utilized for genetic analyses. In such instances, simplifying assumptions could be made with respect to the structure of the data. Some of the assumptions commonly used are:

- (i) Parents have no records of their own. If some parents actually have records, these are ignored. Thus, the **RAM** becomes a **sire-dam model (SDM)**.
- (ii) Parents have no records and dams are related only through their sires. Here, the **RAM** becomes a **sire-maternal grandsire model (SMM)**.
- (iii) Parents have no records and dams are unrelated. Then, the **RAM** becomes a **sire model (SM)**.

Sire-Dam Model (SDM)

Parents have (or assumed to have) no records, thus the **RAM** becomes:

$$y_n = X_n b + Z_n (\frac{1}{2} P_{np}) u_p + (Z_n \varphi_n + e_n)$$

$$y_n = X_n b + Z_n (\frac{1}{2} P_{np})(2)(2^{-1}) u_p + (Z_n \varphi_n + e_n)$$

$$y_n = X_n b + Z_n P_{np} (\frac{1}{2} u_p) + (Z_n \varphi_n + e_n)$$

$$E[y_n] = X_n b$$

$$\text{var} \begin{bmatrix} \frac{1}{2} u_p \\ Z_n \varphi + e_n \end{bmatrix} = \begin{bmatrix} A(\frac{1}{4}\sigma_A^2) & 0 \\ 0 & Z_n D_n Z_n' \sigma_A^2 + I \sigma_e^2 \end{bmatrix}$$

$$= \begin{bmatrix} A\left(\frac{\alpha}{4}\right) & 0 \\ 0 & Z_n D_n Z_n' \alpha + I \end{bmatrix} \sigma_e^2$$

where

y_n = vector of observations of nonparents

b = vector of fixed effects

$\frac{1}{2} u_p$ = vector of transmitting abilities of parents

ϕ_n = vector of deviations of nonparent BV from the average of the BV of their parents
(caused by mendelian sampling)

e_n = vector of nonparent residual effects

X_n = incidence matrix relating nonparent records to fixed effects in b

Z_n = incidence matrix relating nonparent records to individual BV (i.e., to u_{np})

P_{np} = incidence matrix relating nonparent BV (i.e., u_{np}) to the transmitting abilities ($\frac{1}{2}$ BV)

of their parents (i.e., $\frac{1}{2} u_p$)

Let

$$y = y_n, X = X_n, Z = Z_n P_{np}, u = \frac{1}{2} u_p, \text{ and } e = Z_n \phi_n + e_n.$$

All these vectors and matrices are defined above, except for $Z = Z_n P_{np}$, which is an incidence matrix relating nonparent records to the transmitting abilities of the parents of these individuals (i.e., to $u = \frac{1}{2} u_p$).

Thus, the SDM can be expressed as follows:

$$y = Xb + Zu + e$$

$$E[y] = Xb$$

$$\text{var} \begin{bmatrix} u \\ e \end{bmatrix} = \begin{bmatrix} A \left(\frac{\alpha}{4} \right) & 0 \\ 0 & Z_n D_n Z_n' \alpha + I \end{bmatrix} \sigma_e^2$$

To obtain **back solutions for nonparents** we use the same formulae used for the RAM.

Thus,

$$\hat{\phi}_n = (Z_n' Z_n + D^{-1} \alpha^{-1})^{-1} [Z_n' y_n - Z_n' X_n b^o - Z_n' Z \hat{u}]$$

and

$$\hat{u}_n = P_{np} \hat{u} + \hat{\phi}_n$$

For the i^{th} nonparent, these formulae are:

$$\hat{\phi}_{n_i} = \frac{d_{ii}}{n_{\bullet i} d_{ii} + \alpha^{-1}} \left[y_{\bullet i \bullet} - \sum_k n_{ki} b_k \circ \right] - \frac{n_{\bullet i} d_{ii}}{n_{\bullet i} d_{ii} + \alpha^{-1}} [\delta_{s_i} \hat{u}_{s_i} + \delta_{d_i} \hat{u}_{d_i}]$$

and

$$\hat{u}_{n_i} = (\delta_{s_i} \hat{u}_{s_i} + \delta_{d_i} \hat{u}_{d_i}) + \hat{\phi}_{n_i}$$

Remarks:

- [1] Columns of zeroes will be added in front of the matrix $Z_n P_{np} = Z$ for animals needed to obtain the matrix A^{-1} using Henderson's rules.
- [2] The vector u of SDM is equal to the vector $\frac{1}{2} u$ of RAM. This is the reason why the $\frac{1}{2}$'s do not appear in the formulae for $\hat{\phi}_n$ and \hat{u}_n above. **The \hat{u}_n , however, are the BLUP of the BV of the nonparents.**

Numerical example for SDM

Assume that parents in the example for the RAM have no records. Then, the SDM is:

$$\begin{bmatrix} 289 \\ 285 \\ 265 \\ 290 \\ 288 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & | & 1 & 0 & 0 & | & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & | & 0 & 1 & 0 & | & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & | & 0 & 1 & 0 & | & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & | & 0 & 0 & 1 & | & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & | & 0 & 0 & 1 & | & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{bmatrix} + (\phi_n + e_n)$$

The diagonals of the A matrix for the SDM are the same as those of the RAM, thus so are the diagonals of D and D^{-1} . Also, the elements of D_n and D^{-1} are the same for the RAM as for the SDM.

The MME for the SDM are:

$$\begin{bmatrix} X' R_2^{-1} X & X' R_2^{-1} Z \\ Z' R_2^{-1} X & Z' R_2^{-1} Z + A^{-1} \left(\frac{4}{\alpha} \right) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X' R_2^{-1} y \\ Z' R_2^{-1} y \end{bmatrix}$$

where

$$\begin{aligned} R_2^{-1} &= (I_n D_n I_n \alpha + I)^{-1} \\ &= (D_n \alpha + I)^{-1} \end{aligned}$$

$$= \begin{bmatrix} 1.0859375 & & & \\ & 1.09375 & & \\ & & 1.11426781 & \\ & & & 1.11572266 \\ & & & & 1.10473633 \end{bmatrix}^{-1}$$

and

$$\left(\frac{4}{\alpha} \right) = \frac{4}{0.25} = 16$$

The **MME for the SDM** are:

Mauricio A. Elzo, University of Florida, 1996, 2005, 2010.

[17-6]

$$\left[\begin{array}{cccccccccc|c|c}
 2.731 & 0 & 0 & 0.921 & 0.914 & 0.896 & 0.921 & 0.914 & 0 & 0.896 & 0 & b_1 & 786.622 \\
 1.803 & | & 0 & 0 & 0 & 0.898 & 0.905 & 0 & 0 & 0.898 & 0 & 0.905 & b_2 & 498.522 \\
 \hline
 & | & 40.0 & -2.667 & -16.0 & -10.667 & -10.667 & 0 & 0 & 0 & 0 & 0 & u_1 & 0 \\
 & | & & 38.476 & -6.857 & 0 & 0 & -18.286 & 0 & 0 & 0 & 0 & u_2 & 0 \\
 \hline
 & | & & & 53.7 & 0 & 0 & -5.729 & -23.273 & 0 & 0 & 0 & u_3 & 266.130 \\
 & | & & & & 33.812 & 0 & 0 & 11.581 & -20.436 & 0 & 0 & u_4 & = 498.398 \\
 & | & & & & & 40.528 & 0 & 0 & 8.752 & -7.966 & -16.378 & u_5 & 520.617 \\
 \hline
 & | & Symmetric & & & & 49.129 & -23.273 & 0 & 0 & 0 & 0 & u_6 & 266.130 \\
 & | & & & & & 58.126 & -21.333 & 0 & 0 & 0 & 0 & u_7 & 260.571 \\
 & | & & & & & & 52.316 & -17.504 & 0 & 0 & 0 & u_8 & 237.827 \\
 & | & & & & & & & 44.546 & -17.283 & 0 & 0 & u_9 & 259.921 \\
 & | & & & & & & & & 35.471 & 0 & 0 & u_{10} & 260.696
 \end{array} \right]$$

The **vector of solutions for the SDM** is:

$$\begin{bmatrix} b_1^\circ \\ b_2^\circ \\ \vdots \\ \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \\ \hat{u}_4 \\ \hat{u}_5 \\ \hat{u}_6 \\ \hat{u}_7 \\ \hat{u}_8 \\ \hat{u}_9 \\ \hat{u}_{10} \end{bmatrix} = \begin{bmatrix} 288.0682 \\ 276.4162 \\ \vdots \\ -0.0338 \\ -0.1015 \\ -0.1354 \\ -0.6835 \\ 0.7850 \\ -0.1579 \\ -0.2572 \\ -0.6203 \\ 0.2353 \\ 0.7727 \end{bmatrix}$$

where the \hat{u}_i are transmitting abilities, i.e., $\frac{1}{2}$ BV.

Backsolutions can be computed for the nonparents (i.e., animals 11 through 15), using the following formulae for the i^{th} nonparent:

$$\hat{\phi}_{n_i} = \frac{d_{ii}}{d_{ii} + \alpha^{-1}} [y_i - b_i^\circ - (u_{s_i} + u_{d_i})]$$

where

\hat{u}_{s_i} and \hat{u}_{d_i} are from the SDM, and

$$\hat{u}_{n_i} = (u_{s_i} + u_{d_i}) + \hat{\phi}_{n_i}$$

Thus,

$$\begin{bmatrix} \hat{\phi}_{11} \\ \hat{\phi}_{12} \\ \hat{\phi}_{13} \\ \hat{\phi}_{14} \\ \hat{\phi}_{15} \end{bmatrix} = \begin{bmatrix} 0.0791[289 - 288.0682 - (-0.1354 - 0.1579)] \\ 0.0857[285 - 288.0682 - (-0.6835 - 0.2572)] \\ 0.1025[265 - 276.4162 - (-0.6835 - 0.6203)] \\ 0.1037[290 - 288.0682 - (0.7850 + 0.2353)] \\ 0.0948[288 - 276.4162 - (0.7850 + 0.7727)] \end{bmatrix} = \begin{bmatrix} 0.0969 \\ -0.1823 \\ -1.3652 \\ 0.0945 \\ 0.9505 \end{bmatrix}$$

and

$$\begin{bmatrix} \hat{u}_{11} \\ \hat{u}_{12} \\ \hat{u}_{13} \\ \hat{u}_{14} \\ \hat{u}_{15} \end{bmatrix} = \begin{bmatrix} -0.2933 + 0.0969 \\ -0.9407 - 0.1823 \\ -1.3038 - 1.3652 \\ 1.0203 + 0.0945 \\ 1.5577 + 0.9505 \end{bmatrix} = \begin{bmatrix} -0.1964 \\ -1.1230 \\ -2.3403 \\ 1.1148 \\ 2.5082 \end{bmatrix}$$

Sire-Maternal Grandsire Model (SMM)

Dams are assumed to have no records and to be unrelated to other dams and to sires, except through the relationships that may exist among their sires. Under these assumptions the **SDM** is modified as follows:

$$y_n = X_n b + Z_n [P_{ns} \quad P_{nd}] \begin{bmatrix} \frac{1}{2} u_s \\ \frac{1}{2} u_d \end{bmatrix} + (Z_n \phi_n + e_n)$$

$$y_n = X_n b + Z_n [P_{ns} \quad \frac{1}{2} P_{n\text{mgs}} \quad \frac{1}{2} P_{n\text{mgd}} \quad \frac{1}{2} I] \begin{bmatrix} \frac{1}{2} u_s \\ \frac{1}{2} u_{\text{mgs}} \\ \frac{1}{2} u_{\text{mgd}} \\ \phi_d \end{bmatrix} + (Z_n \phi_n + e_n)$$

$$y_n = X_n b + Z_n [P_{ns} - \frac{1}{2} P_{n mgs}] \begin{bmatrix} \frac{1}{2} u_s \\ \frac{1}{2} u_{mgs} \end{bmatrix} + [Z_n P_{n mgd} (\frac{1}{4} u_{mgs}) + Z_n (\frac{1}{2} \phi_d) + Z_n \phi_n + e_n]$$

$$E[y_n] = X_n b$$

$$\text{var} \begin{bmatrix} \frac{1}{2} u_s \\ \frac{1}{2} u_{mgs} \\ \vdots \\ Z_n P_{n mgd} (\frac{1}{4} u_{mgs}) \\ + Z_n (\frac{1}{2} \phi_d + \phi_n) + e_n \end{bmatrix} = \begin{bmatrix} A_{ss} \left(\frac{\alpha}{4} \right) & A_{s mgs} \left(\frac{\alpha}{4} \right) & | & 0 \\ A_{mgs s} \left(\frac{\alpha}{4} \right) & A_{mgs mgs} \left(\frac{\alpha}{4} \right) & | & 0 \\ \vdots & \vdots & | & \vdots \\ 0 & 0 & | & Z_n P_{n mgd} I P_{n mgd}' Z_n' \left(\frac{\alpha}{16} \right) \\ & & | & + Z_n (D_d + D_n) Z_n' \alpha + I \end{bmatrix} \sigma_e^2$$

However, **if nonparents are related only through their male ancestors**, then

$$P_{n mgd} = I,$$

which implies that **each nonparent has a different maternal granddam**. Thus,

$$\text{var} \begin{bmatrix} \frac{1}{2} u_s \\ \frac{1}{2} u_{mgs} \\ \vdots \\ Z_n I (\frac{1}{4} u_{mgs}) + Z_n (\frac{1}{2} \phi_d + \phi_n) + e_n \end{bmatrix} = \begin{bmatrix} A_{ss} \left(\frac{\alpha}{4} \right) & A_{s mgs} \left(\frac{\alpha}{4} \right) & | & 0 \\ A_{mgs s} \left(\frac{\alpha}{4} \right) & A_{mgs mgs} \left(\frac{\alpha}{4} \right) & | & 0 \\ \vdots & \vdots & | & \vdots \\ 0 & 0 & | & Z_n D Z_n' \alpha + I \end{bmatrix} \sigma_e^2$$

where

$\frac{1}{2} u_{mgs}$ = transmitting ability of the maternal grandsire,

$\frac{1}{2} u_{mgd}$ = transmitting ability of the maternal granddam,

$P_{n mgs}$ = incidence matrix relating nonparents to mgs,

$P_{n \text{ mgd}} = I$ = incidence matrix relating nonparents to mgd, and

$\frac{1}{2} \varphi_d = \frac{1}{4} \varepsilon_{mgs} + \frac{1}{4} \varepsilon_{mgd} =$ mendelian sampling in the mgs and the mgd.

All the other vectors and matrices are as defined for the SDM.

Let

$$y = y_n, \quad X = X_n, \quad u = \begin{bmatrix} \frac{1}{2} u_s \\ \frac{1}{2} u_{mgs} \end{bmatrix},$$

$Z = Z_n [P_{ns} \quad P_{n \text{ mgd}}]$ = incidence matrix that relates nonparent records to the nonparent sire and mgs transmitting abilities, and

$$e = Z_n (\frac{1}{4} u_{mgd} + \frac{1}{2} \varphi_d + \varphi_n) + e_n.$$

Then, the SMM is:

$$y = Xb + Zu + e$$

$$E[y] = Xb$$

$$\text{var} \begin{bmatrix} u \\ \dots \\ e \end{bmatrix} = \begin{bmatrix} A\left(\frac{\alpha}{4}\right) & | & 0 \\ \hline \dots & | & \dots \\ 0 & | & Z_n D Z_n' \alpha + I \end{bmatrix} \sigma_e^2$$

$$= \begin{bmatrix} A\left(\frac{\alpha}{4}\right) & 0 \\ 0 & R \end{bmatrix} \sigma_e^2$$

The MME for the SMM are:

$$\begin{bmatrix} X' R^{-1} X & X' R^{-1} Z \\ Z' R^{-1} X & Z' R^{-1} Z + A^{-1} \left(\frac{4}{\alpha} \right) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X' R^{-1} y \\ Z' R^{-1} y \end{bmatrix}$$

How to obtain **back solutions for dams and nonparents?**

Rewrite the SMM as follows:

$$\mathbf{y} = \mathbf{X}\mathbf{b} + [\mathbf{Z} \quad \mathbf{I}] \begin{bmatrix} \mathbf{u} \\ \mathbf{e} \end{bmatrix}$$

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{W}\boldsymbol{\theta}, \text{ for } \mathbf{W} = [\mathbf{Z} \quad \mathbf{I}] \text{ and } \boldsymbol{\theta} = \begin{bmatrix} \mathbf{u} \\ \mathbf{e} \end{bmatrix}.$$

$$E[\mathbf{y}] = \mathbf{X}\mathbf{b}$$

$$\text{var}(\boldsymbol{\theta}) = \mathbf{B}$$

$$\Rightarrow \text{var}(\mathbf{y}) = \mathbf{WBW}'$$

$$\Rightarrow \text{cov}(\boldsymbol{\theta}, \mathbf{y}') = \text{cov}(\boldsymbol{\theta}, \boldsymbol{\theta}'\mathbf{W}')$$

$$= \mathbf{BW}'$$

\Rightarrow BLUP of $\boldsymbol{\theta}$ is, by definition,

$$\hat{\boldsymbol{\theta}} = \mathbf{BW}'(\mathbf{WBW}')^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b}^0)$$

Suppose we want to compute the BLUP of γ , and we know that:

$$E[\gamma] = 0$$

$$\text{cov}(\gamma, \boldsymbol{\theta}') = \mathbf{C}$$

$$\Rightarrow \text{cov}(\gamma, \mathbf{y}') = \mathbf{CW}'$$

\Rightarrow the BLUP of γ is:

$$\hat{\gamma} = \mathbf{CW}'(\mathbf{WBW}')^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b}^0)$$

$$\hat{\gamma} = \mathbf{CB}^{-1} \mathbf{BW}'(\mathbf{WBW}')^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b}^0)$$

$$\Rightarrow \hat{\gamma} = \mathbf{CB}^{-1} \hat{\boldsymbol{\theta}}$$

where

$\mathbf{C}\mathbf{B}^{-1}$ = regression of γ on θ , and

$$\hat{\theta} = \mathbf{B}\mathbf{W}'(\mathbf{W}\mathbf{B}\mathbf{W}')^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b}^0).$$

In many instances,

$$\mathbf{B} = \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}$$

Also,

$$\mathbf{C} = [\mathbf{C}_u \ \mathbf{C}_e]$$

=> the BLUP of γ is:

$$\hat{\gamma} = \mathbf{C}_u \mathbf{G}^{-1} \hat{u} + \mathbf{C}_e \mathbf{R}^{-1} \hat{e}$$

In the SMM:

$$\theta = \begin{bmatrix} \mathbf{u} \\ \mathbf{e} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{A}\left(\frac{\alpha}{4}\right) & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \sigma_e^2$$

$$\Rightarrow \mathbf{B}^{-1} = \begin{bmatrix} \mathbf{A}^{-1}\left(\frac{4}{\alpha}\right) & \mathbf{0} \\ \mathbf{0} & (\mathbf{Z}_n \mathbf{D} \mathbf{Z}_n' + \mathbf{I})^{-1} \end{bmatrix} \sigma_e^{-2}$$

$$\gamma = \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_n \end{bmatrix}$$

$$\mathbf{C}_u = \text{cov}(\gamma, \mathbf{u}')$$

$$\mathbf{C}_u = \text{cov}\left\{ \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_n \end{bmatrix}, \mathbf{u}' \right\}$$

$$C_u = \text{cov} \left\{ \begin{bmatrix} u_d \\ u_s \\ u_n \end{bmatrix}, \begin{bmatrix} \frac{1}{2} u_s' \\ \frac{1}{2} u_{mgs}' \\ \frac{1}{2} u_{mgs}' \end{bmatrix} \right\}$$

where

$$\begin{aligned} \text{cov}(u_d, \frac{1}{2} u_s') &= \text{cov}(\frac{1}{2} P_{d mgs} u_{mgs} + (\frac{1}{2} u_{mgd} + \phi_d), \frac{1}{2} u_s') \\ &= (\frac{1}{4} P_{d mgs} A_{mgs, mgs} \alpha) \sigma_e^2 \\ \text{cov}(u_d, \frac{1}{2} u_{mgs}') &= (\frac{1}{2} P_{d mgs} A_{mgs, mgs} \alpha) \sigma_e^2 \\ \text{cov}(u_n, \frac{1}{2} u_s') &= \text{cov}(\frac{1}{2} P_{ns} u_s + \frac{1}{4} P_{n mgs} u_{mgs} + \frac{1}{4} P_{n mgd} u_{mgd} + \frac{1}{2} \phi_d + \phi_n, \frac{1}{2} u_s') \\ &= (\frac{1}{4} P_{ns} A_{ss} \alpha + \frac{1}{8} P_{n mgs} A_{mgs, s} \alpha) \sigma_e^2 \\ \text{cov}(u_n, \frac{1}{2} u_{mgs}') &= (\frac{1}{4} P_{ns} A_{s, mgs} \alpha + \frac{1}{8} P_{n mgs} A_{mgs, mgs} \alpha) \sigma_e^2 \\ C_e &= \text{cov} \left\{ \begin{bmatrix} u_d \\ u_s \\ u_n \end{bmatrix}, e' \right\} \\ C_e &= \text{cov} \left\{ \begin{bmatrix} u_d \\ u_n \end{bmatrix}, e_n' + (\phi_n' + \frac{1}{2} \phi_d' + \frac{1}{4} u_{mgd}', Z_n') \right\} \end{aligned}$$

where

$$\begin{aligned} \text{cov}(u_d, e') &= \text{cov}(\frac{1}{2} P_{d mgs} u_{mgs} + (\frac{1}{2} u_{mgd} + \phi_d), e_n' + (\frac{1}{2} \phi_d' + \phi_n' + \frac{1}{4} u_{mgs}', Z_n')) \\ &= (\frac{1}{8} A_{mgd, mgd} Z_n' \alpha + \frac{1}{2} D_d Z_n' \alpha) \sigma_e^2 \\ &= [(\frac{1}{8} I + \frac{1}{2} D_d) Z_n' \alpha] \sigma_e^2 \\ &= [D_l Z_n' \alpha] \sigma_e^2 \\ \text{cov}(u_n, e') &= \text{cov}(\frac{1}{2} P_{ns} u_s + \frac{1}{4} P_{n mgs} u_{mgs} + \frac{1}{4} P_{n mgd} u_{mgd} + \frac{1}{2} \phi_d + \phi_n, \\ &\quad e_n' + (\frac{1}{2} \phi_d' + \phi_n' + \frac{1}{4} u_{mgd}', Z_n')) \\ &= (1/16 P_{n mgd} A_{mgd, mgd} Z_n' \alpha + (\frac{1}{4} D_d + D_n) Z_n' \alpha) \sigma_e^2 \\ &= [(1/16 I + \frac{1}{4} D_d + D_n) Z_n' \alpha] \sigma_e^2 \end{aligned}$$

Mauricio A. Elzo, University of Florida, 1996, 2005, 2010.

[17-14]

$$= [D_2 Z_n' \alpha] \sigma_e^2$$

where

D_2 = diagonal matrix whose entries are:

$$d_{ii} = \left[1 - \delta_{s_i} \left(\frac{1}{4} a_{s_i s_i} \right) - \delta_{mgs_i} \left(\frac{1}{16} a_{mgs_i mgs_i} \right) \right]$$

where

$$\delta_{s_i} = \begin{cases} 1 & \text{if } s_i \text{ is known} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{mgs_i} = \begin{cases} 1 & \text{if } mgs_i \text{ is known} \\ 0 & \text{otherwise} \end{cases}$$

D_1 = diagonal matrix whose entries are equal to:

$$d_{ii} = \frac{1}{2} \left[1 - \delta_{mgs_i} \left(\frac{1}{4} a_{mgs_i mgs_i} \right) \right]$$

The covariance matrix C_u , ordered by mgs' first and then sires, is:

$$C_u = \begin{bmatrix} \frac{1}{4} P_{d mgs} A_{mgs, mgs} \alpha & | & \frac{1}{4} P_{d mgs} A_{mgs, s} \alpha \\ 1/8 P_{n mgs} A_{mgs, mgs} \alpha + \frac{1}{4} P_{ns} A_{s, s} \alpha & | & 1/8 P_{n mgs} A_{mgs, s} \alpha + \frac{1}{4} P_{ns} A_{s, s} \alpha \end{bmatrix} \sigma_e^2$$

The covariance matrix C_e is:

$$C_e = \begin{bmatrix} \left(\frac{1}{8} I + \frac{1}{2} D_d \right) Z_n' \alpha \\ \left(\frac{1}{16} I + \frac{1}{4} D_d + D_n \right) Z_n' \alpha \end{bmatrix} \sigma_e^2$$

$$C_e = \begin{bmatrix} D_l Z_n' \alpha \\ D_2 Z_n' \alpha \end{bmatrix} \sigma_e^2$$

The matrix G^{-1} is:

$$G^{-1} = \begin{bmatrix} A_{s,s} \left(\frac{4}{\alpha} \right) & A_{s,mgs} \left(\frac{4}{\alpha} \right) \\ A_{mgs,s} \left(\frac{4}{\alpha} \right) & A_{mgs,mgs} \left(\frac{4}{\alpha} \right) \end{bmatrix} \sigma_e^{-2}$$

The matrix R^{-1} is:

$$R^{-1} = (Z_n D_2 Z_n' \alpha + I)^{-1} \sigma_e^{-2}$$

$$\hat{u} = \begin{bmatrix} \frac{1}{2} \hat{u}_s \\ \frac{1}{2} \hat{u}_{mgs} \end{bmatrix}$$

Thus, the **BLUP of e and γ for the SMM are:**

$$\hat{e} = (y - X b^o - Z \hat{u})$$

$$\hat{\gamma} = C_u G^{-1} \hat{u} + C_e R^{-1} \hat{e}$$

where

$$\begin{aligned} C_u G^{-1} \hat{u} &= \begin{bmatrix} \frac{\alpha}{4} P_{d,mgs} & 0 \\ \frac{\alpha}{8} P_{n,mgs} & \frac{\alpha}{4} P_{n,s} \end{bmatrix} \begin{bmatrix} A_{mgs,mgs} & A_{mgs,s} \\ A_{s,mgs} & A_{s,s} \end{bmatrix} \begin{bmatrix} A_{mgs,mgs} & A_{mgs,s} \\ A_{s,mgs} & A_{s,s} \end{bmatrix} \left(\frac{4}{\alpha} \right) \begin{bmatrix} \frac{1}{2} \hat{u}_{mgs} \\ \frac{1}{2} \hat{u}_s \end{bmatrix} \\ &= \begin{bmatrix} P_{d,mgs} & 0 \\ \frac{1}{2} P_{n,mgs} & P_{n,s} \end{bmatrix} \begin{bmatrix} \frac{1}{2} \hat{u}_{mgs} \\ \frac{1}{2} \hat{u}_s \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2} P_{d,mgs} \hat{u}_{mgs} \\ \frac{1}{4} P_{n,mgs} \hat{u}_{mgs} + \frac{1}{2} P_{n,s} \hat{u}_s \end{bmatrix} \\ C_e R^{-1} \hat{e} &= \begin{bmatrix} D_1 Z_n' \alpha \\ D_2 Z_n' \alpha \end{bmatrix} (Z_n D_2 Z_n' \alpha + I)^{-1} [y - X b^o - Z_n P_{n,mgs} (\frac{1}{4} \hat{u}_{mgs}) - Z_n P_{n,s} (\frac{1}{2} \hat{u}_s)] \end{aligned}$$

Thus, the elements of $\hat{\gamma}$, i.e., \hat{u}_d and \hat{u}_n , are equal to:

$$\hat{u}_d = \frac{1}{2} P_{d_{mgs}} \hat{u}_{mgs} + D_1 Z_n' \alpha (Z_n D_2 Z_n' \alpha + I)^{-1} [y - X b^o - Z_n P_{n_{mgs}} (\frac{1}{4} \hat{u}_{mgs}) - Z_n P_{ns} (\frac{1}{2} \hat{u}_s)]$$

and

$$\hat{u}_n = \frac{1}{4} P_{n_{mgs}} \hat{u}_{mgs} + \frac{1}{2} P_{ns} \hat{u}_s + D_2 Z_n' \alpha (Z_n D_2 Z_n' \alpha + I)^{-1} [y - X b^o - Z_n P_{n_{mgs}} (\frac{1}{4} \hat{u}_{mgs}) - Z_n P_{ns} (\frac{1}{2} \hat{u}_s)]$$

Let $Z_n = I$, i.e., there is **only one record per nonparent**. Then,

$$\hat{u}_d = P_{d_{mgs}} (\frac{1}{2} \hat{u}_{mgs}) + D_1 \alpha (D_2 \alpha + I)^{-1} [y - X b^o - P_{n_{mgs}} (\frac{1}{4} \hat{u}_{mgs}) - P_{ns} (\frac{1}{2} \hat{u}_s)]$$

and

$$\hat{u}_n = P_{n_{mgs}} (\frac{1}{4} \hat{u}_{mgs}) + P_{ns} (\frac{1}{2} \hat{u}_s) + D_2 \alpha (D_2 \alpha + I)^{-1} [y - X b^o - P_{n_{mgs}} (\frac{1}{4} \hat{u}_{mgs}) - P_{ns} (\frac{1}{2} \hat{u}_s)]$$

Hence, if $Z_n = I$, the BLUP of the BV of the i^{th} dam (d_i) and the i^{th} nonparent (n_i) are:

$$\begin{aligned} \hat{u}_{d_i} &= \frac{1}{2} \hat{u}_{mgs_i} + \frac{d_{1ii} \alpha}{d_{2ii} \alpha + 1} [y_i - b_i^o - \frac{1}{4} \hat{u}_{mgs_i} - \frac{1}{2} \hat{u}_{s_i}] \\ &\quad \downarrow \qquad \qquad \downarrow \\ &\quad \text{pedigree contribution} \qquad \text{data contribution} \end{aligned}$$

where

d_{1ii} = element of D_1 (**matrix D for the dam**)

d_{2ii} = element of D_2 (**matrix D for the calf**)

and

$$\begin{aligned} \hat{u}_{n_i} &= \frac{1}{4} \hat{u}_{mgs_i} + \frac{1}{2} \hat{u}_{s_i} + \frac{d_{2ii} \alpha}{d_{2ii} \alpha + 1} [y_i - b_i^o - \frac{1}{4} \hat{u}_{mgs_i} - \frac{1}{2} \hat{u}_{s_i}] \\ &\quad \downarrow \qquad \qquad \downarrow \\ &\quad \text{pedigree contribution} \qquad \text{data contribution} \end{aligned}$$

Numerical example for SMM

The **SMM for the example given in the RAM section** is:

$$\begin{bmatrix} 289 \\ 285 \\ 265 \\ 290 \\ 288 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} 0 & | & 1.5 & 0 & 0 \\ 0 & | & 0.5 & 1 & 0 \\ 0 & | & 0 & 1.5 & 0 \\ 0 & | & 0 & 0 & 1.5 \\ 0 & | & 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} + (\frac{1}{4} u_{mgd} + \frac{1}{2} \phi_d + \phi_n + e_n)$$

The **MME** are:

$$\begin{bmatrix} X'R_2^{-1}X & X'R_2^{-1}Z \\ Z'R_2^{-1}X & Z'R_2^{-1}X + A^{-1}\left(\frac{4}{\alpha}\right) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X'R_2^{-1}y \\ Z'R_2^{-1}y \end{bmatrix}$$

where

$$\left(\frac{4}{\alpha}\right) = \frac{4}{0.25} = 16$$

$$R_2^{-1} = (D\alpha + I)^{-1}$$

$$R_2^{-1} = \begin{bmatrix} 1.15234375 & & & & \\ & 1.16796875 & & & \\ & & 1.171875 & & \\ & & & 1.171875 & \\ & & & & 1.171875 \end{bmatrix}^{-1}$$

The **inverse of the relationship matrix among sires and maternal grandsires** is:

$$A^{-1} = \begin{bmatrix} 2.4849 & -1.0909 & -0.6667 & -0.6667 \\ & 1.4546 & 0 & 0 \\ \text{Symmetric} & & 1.3333 & 0 \\ & & & 1.3333 \end{bmatrix}$$

The **MME for the SMM** are:

$$\left[\begin{array}{ccc|ccccc} 2.577 & 0 & 0 & 1.730 & 0.856 & 1.280 & b_1 & 742.273 \\ & 1.707 & 0 & 0 & 1.280 & 1.280 & b_2 & 471.893 \\ \hline \hline & & & 39.758 & -17.455 & -10.667 & u_1 & \dots \\ & & & & 25.439 & 0.428 & u_2 & 498.197 \\ \text{Symmetric} & & & & 24.110 & 0 & u_3 & 583.213 \\ & & & & & 25.173 & u_4 & 739.840 \end{array} \right] = \begin{bmatrix} b_1 \\ b_2 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$

The **vector of solutions for the SMM** is:

$$\begin{bmatrix} b_1 \\ b_2 \\ \hat{u}_1 \\ \hat{u}_3 \\ \hat{u}_4 \\ \hat{u}_5 \end{bmatrix} = \begin{bmatrix} 287.8753 \\ 276.5113 \\ 0.0075 \\ 0.0263 \\ -0.7104 \\ 0.6954 \end{bmatrix}$$

To obtain **backsolutions for dams and nonparents we need the d_{1ii} and the d_{2ii}** . The d_{1ii} are computed for dams 6 to 10, and the d_{2ii} for nonparents 11 to 15.

Thus, for **dams 6 to 10** we compute:

| i | $(d_{l_{ii}})$ | $(d_{l_{ii}} \alpha)$ |
|----|-----------------------|-----------------------|
| 6 | $\frac{1}{2}(0.75)$ | 0.09375 |
| 7 | $\frac{1}{2}(0.6875)$ | 0.0859375 |
| 8 | $\frac{1}{2}(0.6875)$ | 0.0859375 |
| 9 | $\frac{1}{2}(0.75)$ | 0.09375 |
| 10 | $\frac{1}{2}(0.75)$ | 0.09375 |

and for **nonparents 11 to 15** we compute :

| i | $(d_{2_{ii}})$ | $(d_{2_{ii}} \alpha)$ |
|----|----------------|-----------------------|
| 11 | 0.609375 | 0.15234375 |
| 12 | 0.671875 | 0.16796875 |
| 13 | 0.6875 | 0.171875 |
| 14 | 0.6875 | 0.171875 |
| 15 | 0.6875 | 0.171875 |

The **backsolutions for dams 6 to 10** are:

$$\begin{bmatrix} \hat{u}_6 \\ \hat{u}_7 \\ \hat{u}_8 \\ \hat{u}_9 \\ \hat{u}_{10} \end{bmatrix} = \begin{bmatrix} (0.0263) + 0.081356[289 - 287.875 - (\frac{1}{2}(0.0263) + 0.0263)] \\ (0.0263) + 0.073579[285 - 287.875 - (\frac{1}{2}(-0.7104) + 0.0263)] \\ (-0.7104) + 0.073333[265 - 276.511 - (\frac{1}{2}(-0.7104) - 0.7104)] \\ (0.6954) + 0.080000[290 - 287.875 - (\frac{1}{2}(0.6954) + 0.6954)] \\ (0.6954) + 0.080000[288 - 276.511 - (\frac{1}{2}(0.6954) + 0.6954)] \end{bmatrix}$$

$$\begin{bmatrix} \hat{u}_6 \\ \hat{u}_7 \\ \hat{u}_8 \\ \hat{u}_9 \\ \hat{u}_{10} \end{bmatrix} = \begin{bmatrix} (0.0263) + 0.081356[289 - 287.875 - (0.0395)] \\ (0.0263) + 0.073579[285 - 287.875 - (-0.3289)] \\ (-0.7104) + 0.073333[265 - 276.511 - (-1.0656)] \\ (0.6954) + 0.080000[290 - 287.875 - (1.0431)] \\ (0.6954) + 0.080000[288 - 276.511 - (1.0431)] \end{bmatrix}$$

$$\begin{bmatrix} \hat{u}_6 \\ \hat{u}_7 \\ \hat{u}_8 \\ \hat{u}_9 \\ \hat{u}_{10} \end{bmatrix} = \begin{bmatrix} (0.0263) + 0.081356[1.4605] \\ (0.0263) + 0.073579[-2.5461] \\ (-0.7104) + 0.073333[-10.4454] \\ (0.6954) + 0.080000[1.0819] \\ (0.6954) + 0.080000[10.4459] \end{bmatrix}$$

$$\begin{bmatrix} \hat{u}_6 \\ \hat{u}_7 \\ \hat{u}_8 \\ \hat{u}_9 \\ \hat{u}_{10} \end{bmatrix} = \begin{bmatrix} 0.1451 \\ -0.1610 \\ -1.4764 \\ 0.7820 \\ 1.5311 \end{bmatrix}$$

The **backsolutions for nonparents 11 to 15** are:

$$\begin{bmatrix} \hat{u}_{11} \\ \hat{u}_{12} \\ \hat{u}_{13} \\ \hat{u}_{14} \\ \hat{u}_{15} \end{bmatrix} = \begin{bmatrix} (0.0395) + 0.132203[1.4605] \\ (-0.3289) + 0.143813[-2.5461] \\ (-1.0656) + 0.1466667[-10.4454] \\ (1.0431) + 0.1466667[1.0819] \\ (1.0431) + 0.1466667[10.4459] \end{bmatrix}$$

$$\begin{bmatrix} \hat{u}_{11} \\ \hat{u}_{12} \\ \hat{u}_{13} \\ \hat{u}_{14} \\ \hat{u}_{15} \end{bmatrix} = \begin{bmatrix} 0.2326 \\ -0.6951 \\ -2.5976 \\ 1.2018 \\ 2.5752 \end{bmatrix}$$

The **backsolution for dam 2** is:

$$\hat{u}_2 = \text{cov}(u_2, \frac{1}{2}u_s') G^{-1} \hat{u} + \text{cov}(u_2, e') R^{-1} \hat{e}$$

$$\hat{u}_2 = \frac{\frac{1}{4}a_{11}\alpha\sigma_e^2}{a_{11}\left(\frac{\alpha}{4}\right)\sigma_e^2} (\frac{1}{2}\hat{u}_l) + 0 R^{-1} \hat{e}$$

$$\hat{u}_2 = \frac{1}{2}\hat{u}_l$$

$$\hat{u}_2 = 0.0038$$

Remarks:

$$(i) \text{ cov}(u_2, e') = \text{cov}(u_l + \phi_2, e_n' + \frac{1}{2}\phi_d' + \phi_n' + \frac{1}{2}u_{mgd}',) \\ = 0$$

$$(ii) \text{ cov}(u_2, \frac{1}{2}u_s') = \text{cov}(\frac{1}{2}u_l + \phi_2, [\frac{1}{2}u_1 \ \frac{1}{2}u_3 \ \frac{1}{2}u_4 \ \frac{1}{2}u_5]) \\ = [1 \ 0.75 \ 0.5 \ 0.5] \left(\frac{\alpha}{4}\right) \sigma_e^2 \\ = \text{1}^{\text{st}} \text{ row of } G \\ = A \left(\frac{\alpha}{4}\right) \sigma_e^2$$

$$(iii) \text{cov}(\mathbf{u}_2, \frac{1}{2}\mathbf{u}_s) \text{var}(\frac{1}{2}\mathbf{u}_s) = [1 \ 0.75 \ 0.5 \ 0.5] \begin{bmatrix} 2.485 & -1.091 & -0.667 & -0.667 \\ -1.091 & 1.455 & 0 & 0 \\ -0.667 & 0 & 1.333 & 0 \\ -0.667 & 0 & 0 & 1.333 \end{bmatrix}$$

$$= [1 \ 0 \ 0 \ 0]$$

$$(iv) \text{cov}(\mathbf{u}_2, \frac{1}{2}\mathbf{u}_s) \text{var}(\frac{1}{2}\mathbf{u}_s) \hat{\mathbf{u}} = [1 \ 0 \ 0 \ 0] \begin{bmatrix} \frac{1}{2}\hat{u}_1 \\ \frac{1}{2}\hat{u}_3 \\ \frac{1}{2}\hat{u}_4 \\ \frac{1}{2}\hat{u}_5 \end{bmatrix}$$

$$= \frac{1}{2}\hat{u}_1$$

Sire model (SM)

When parents have no records and dams are unrelated, the **RAM** can be written as:

$$\mathbf{y}_n = \mathbf{X}_n \mathbf{b} + \mathbf{Z}_n [\mathbf{P}_{ns} : \mathbf{P}_{nd}] \begin{bmatrix} \frac{1}{2}\mathbf{u}_s \\ \frac{1}{2}\mathbf{u}_d \end{bmatrix} + (\mathbf{Z}_n \boldsymbol{\phi}_n + \mathbf{e}_n)$$

$$\mathbf{y}_n = \mathbf{X}_n \mathbf{b} + \mathbf{Z}_n \mathbf{P}_{ns} \left(\frac{1}{2}\mathbf{u}_s \right) + [\mathbf{Z}_n \mathbf{P}_{nd} \left(\frac{1}{2}\mathbf{u}_d \right) + \mathbf{Z}_n \boldsymbol{\phi}_n + \mathbf{e}_n]$$

But $\mathbf{P}_{nd} = \mathbf{I}$ under the assumption that nonparents are related only through their sires (i.e., each nonparent is assumed to have a different dam). Thus, the **SM** is:

$$\mathbf{y}_n = \mathbf{X}_n \mathbf{b} + \mathbf{Z}_n \mathbf{P}_{ns} \left(\frac{1}{2}\mathbf{u}_s \right) + [\mathbf{Z}_n \left(\frac{1}{2}\mathbf{u}_d \right) + \mathbf{Z}_n \boldsymbol{\phi}_n + \mathbf{e}_n]$$

$$E[\mathbf{y}_n] = \mathbf{X}_n \mathbf{b}$$

$$\text{var} \begin{bmatrix} \frac{1}{2} u_s \\ Z_n (\frac{1}{2} u_d + \phi_n) + e_n \end{bmatrix} - \begin{bmatrix} A_{ss} \left(\frac{\alpha}{4} \right) & | & 0 \\ | & | & | \\ 0 & | & Z_n D Z_n' \alpha + I \end{bmatrix} \sigma_e^2$$

Letting

$y = y_n$, $Z = Z_n P_{ns}$, $u = \frac{1}{2}u_s$, $e = Z_n (\frac{1}{2}u_d + \phi_n) + e_n$, and $R_2 = (Z_n D Z_n' \alpha + I)$, the **SM**

becomes:

$$y = Xb + Zu + e$$

$$E[y] = Xb$$

$$\text{var} \begin{bmatrix} u \\ e \end{bmatrix} = \begin{bmatrix} A_{ss} \left(\frac{\alpha}{4} \right) & 0 \\ 0 & R_2 \end{bmatrix} \sigma_e^2$$

with MME:

$$\begin{bmatrix} X' R_2^{-1} X & X' R_2^{-1} Z \\ Z' R_2^{-1} X & Z' R_2^{-1} Z + (A_{ss})^{-1} \left(\frac{4}{\alpha} \right) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X' R_2^{-1} y \\ Z' R_2^{-1} y \end{bmatrix}$$

Backsolutions for dams and nonparents are obtained as follows:

$$\hat{y} = C_u G^{-1} \hat{u} + C_e R^{-1} \hat{e}$$

where

$$(i) C_u = \text{cov} \left\{ \begin{bmatrix} u_d \\ u_n \end{bmatrix}, \frac{1}{2} u_s \right\}$$

$$\text{Cov}(u_d, \frac{1}{2} u_s) = 0$$

$$\text{cov}(u_n, \frac{1}{2} u_s) = \text{cov}(P_{ns} (\frac{1}{2} u_s) + (\frac{1}{2} u_d + \phi_n), \frac{1}{2} u_s)$$

$$= \left[P_{ns} A_{ss} \left(\frac{\alpha}{4} \right) \right] \sigma_e^2$$

$$\Rightarrow C_u = \begin{bmatrix} 0 \\ P_{ns} A_{ss} \left(\frac{\alpha}{4} \right) \end{bmatrix} \sigma_e^2$$

$$(ii) C_e = \text{cov} \left\{ \begin{bmatrix} u_d \\ u_n \end{bmatrix}, e \right\}$$

$$\text{cov}(u_d, e) = \text{cov} (u_d, e_n + (\frac{1}{2} u_d + \phi_n) Z_n)$$

$$= \left[A_{dd} \left(\frac{\alpha}{2} \right) \right] \sigma_e^2$$

$$= \left[I \left(\frac{\alpha}{2} \right) \right] \sigma_e^2$$

$$\text{cov}(u_n, e) = \text{cov} (P_{ns}(u_s) + (\frac{1}{2} u_d + \phi_n), e_n + (\frac{1}{2} u_d + \phi_n) Z_n)$$

$$= \left[\left(A_{dd} \left(\frac{\alpha}{4} \right) + D_n \alpha \right) Z_n \right] \sigma_e^2$$

$$= \left[\left(I \left(\frac{\alpha}{4} \right) + D_n \alpha \right) Z_n \right] \sigma_e^2$$

$$= [D Z_n \alpha] \sigma_e^2$$

$$(iii) G^{-1} = [\text{var} (\frac{1}{2} u_s)]^l$$

$$= (A_{ss})^l \left(\frac{4}{\alpha} \right) \sigma_e^{-2}$$

$$(iv) R^{-1} = [\text{var} (Z_n (\frac{1}{2} u_d) + \phi_n) + e_n]^l$$

$$= [Z_n D Z_n \alpha + I]^l \sigma_e^{-2}$$

$$(v) \hat{u} = \frac{1}{2} \hat{u}_s$$

$$(vi) \hat{e} = y - Xb^o - Z_n P_{ns} \left(\frac{1}{2} \hat{u}_s \right)$$

$$(vii) C_u G^{-1} \hat{u} = \begin{bmatrix} 0 \\ P_{ns} A_{ss} \left(\frac{\alpha}{4} \right) \end{bmatrix} \left(A_{ss} \right)^{-1} \left(\frac{4}{\alpha} \right) \left(\frac{1}{2} \hat{u}_s \right)$$

$$= \begin{bmatrix} 0 \\ P_{ns} \left(\frac{1}{2} \hat{u}_s \right) \end{bmatrix}$$

$$(viii) C_e R^{-1} \hat{e} = \begin{bmatrix} I \left(\frac{\alpha}{2} \right) \\ DZ_n \alpha \end{bmatrix} \left(Z_n D Z_n \alpha + I \right)^{-1} \left[y - Xb^o - Z_n P_{ns} \left(\frac{1}{2} \hat{u}_s \right) \right]$$

Thus,

$$\hat{u}_d = \left(\frac{\alpha}{2} \right) \left(Z_n D Z_n \alpha + I \right)^{-1} \left[y - Xb^o - Z_n P_{ns} \left(\frac{1}{2} \hat{u}_s \right) \right]$$

and

$$\hat{u}_n = P_{ns} \left(\frac{1}{2} \hat{u}_s \right) + D Z_n \alpha \left(Z_n D Z_n \alpha + I \right)^{-1} \left[y - Xb^o - Z_n P_{ns} \left(\frac{1}{2} \hat{u}_s \right) \right].$$

If nonparents have only one record each, $Z_n = I$. Thus,

$$\hat{u}_d = \left(\frac{\alpha}{2} \right) \left(D \alpha + I \right)^{-1} \left[y - Xb^o - P_{ns} \left(\frac{1}{2} \hat{u}_s \right) \right]$$

and

$$\hat{u}_n = P_{ns} \left(\frac{1}{2} \hat{u}_s \right) + D \alpha \left(D \alpha + I \right)^{-1} \left[y - Xb^o - P_{ns} \left(\frac{1}{2} \hat{u}_s \right) \right].$$

Mauricio A. Elzo, University of Florida, 1996, 2005, 2010.

[17-26]

The BLUP of the BV of the i^{th} dam (d_i) and the i^{th} nonparent (n_i), when $Z_n = I$, are:

$$\hat{u}_{d_i} = \frac{\alpha}{\frac{2}{d_{ii}\alpha+1}} [y_i - b_i^\circ - \frac{1}{2} \hat{u}_{s_i}]$$

↓

data contribution

and

$$\hat{u}_{n_i} = \frac{1}{2} \hat{u}_{s_i} + \frac{d_{ii}\alpha}{d_{ii}\alpha+1} [y_i - b_i^\circ - \frac{1}{2} \hat{u}_{s_i}]$$

↓

↓

pedigree contribution data contribution

Numerical example for SM

The **SM** in the example presented for the **RAM** is:

$$\begin{bmatrix} 289 \\ 285 \\ 265 \\ 290 \\ 288 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} 0 & | & 1 & 0 & 0 \\ 0 & | & 0 & 1 & 0 \\ 0 & | & 0 & 1 & 0 \\ 0 & | & 0 & 0 & 1 \\ 0 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} + \left(\frac{1}{2} u_d + \phi_n + e_n \right)$$

The **MME** are:

$$\begin{bmatrix} X' R_2^{-1} X & X' R_2^{-1} Z \\ Z' R_2^{-1} X & Z' R_2^{-1} Z + A^{-1} \left(\frac{4}{\alpha} \right) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X' R_2^{-1} y \\ Z' R_2^{-1} y \end{bmatrix}$$

where

$$\begin{aligned}
 \left(\frac{4}{\alpha} \right) &= \frac{4}{0.25} \\
 &= 16 \\
 R_2^{-1} &= (D\alpha + I)^{-1} \\
 &= \begin{bmatrix} 1.1875 & & & \\ & 1.1875 & & \\ & & 1.1875 & \\ & & & 1.1875 \end{bmatrix}^{-1} \\
 &= I(1.1875)^{-1}
 \end{aligned}$$

Thus, the MME for the SM can be written as follows:

$$\begin{aligned}
 \begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + A^{-1} \left(\frac{4}{\alpha} \right) (0.75\alpha + 1) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} &= \begin{bmatrix} X'y \\ Z'y \end{bmatrix} \\
 \begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + A^{-1}(19) \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} &= \begin{bmatrix} X'y \\ Z'y \end{bmatrix}
 \end{aligned}$$

The A^{-1} matrix for the SM is:

$$A^{-1} = \begin{bmatrix} 2.0 & -0.6667 & -0.6667 & -0.6667 \\ & 1.333 & 0 & 0 \\ & & 1.333 & 0 \\ & & & 1.333 \end{bmatrix}$$

The MME for the SM are:

$$\left[\begin{array}{ccc|ccccc} 3.0 & 0 & 0 & 1.0 & 1.0 & 1.0 & b_1 & 864 \\ & 2.0 & 0 & 0 & 1.0 & 1.0 & b_2 & 553 \\ \hline - & - & - & - & - & - & - & - \\ & | & 38.0 & -12.667 & -12.667 & -12.667 & u_1 & 0 \\ & | & & 26.333 & 0 & 0 & u_3 & 289 \\ & | & & & 27.333 & 0 & u_4 & 550 \\ & | & & & & 27.333 & u_5 & 578 \end{array} \right] =$$

The vector of solutions for the **SM** is:

$$\begin{bmatrix} b_1^\circ \\ b_2^\circ \\ - \\ \hat{u}_1 \\ \hat{u}_3 \\ \hat{u}_4 \\ \hat{u}_5 \end{bmatrix} = \begin{bmatrix} 288.0000 \\ 276.5189 \\ - \\ 0 \\ 0.0380 \\ -0.5312 \\ 0.4932 \end{bmatrix}$$

The backsolutions for dams 6 to 10 are:

$$\begin{bmatrix} \hat{u}_6 \\ \hat{u}_7 \\ \hat{u}_8 \\ \hat{u}_9 \\ \hat{u}_{10} \end{bmatrix} = \begin{bmatrix} 0.10526 [289 - 288.000 - 0.0380] \\ 0.10526 [285 - 288.000 + 0.5312] \\ 0.10526 [265 - 276.519 + 0.5312] \\ 0.10526 [290 - 288.000 - 0.4932] \\ 0.10526 [288 - 276.519 - 0.4932] \end{bmatrix} = \begin{bmatrix} 0.1013 \\ -0.2600 \\ -1.1566 \\ 0.1586 \\ 1.1566 \end{bmatrix}$$

and the backsolutions for nonparents 11 to 15 are:

$$\begin{bmatrix} \hat{u}_{11} \\ \hat{u}_{12} \\ \hat{u}_{13} \\ \hat{u}_{14} \\ \hat{u}_{15} \end{bmatrix} = \begin{bmatrix} 0.0380 + 0.15789 [0.9620] \\ -0.5312 + 0.15789 [-2.4688] \\ -0.5312 + 0.15789 [-10.9878] \\ 0.4932 + 0.15789 [1.5068] \\ 0.4932 + 0.15789 [10.9878] \end{bmatrix} = \begin{bmatrix} 0.1899 \\ -0.9210 \\ -2.2661 \\ 0.7311 \\ 2.2281 \end{bmatrix}$$

The **backsolution for dam u_2** is:

$$\begin{aligned} \hat{u}_2 &= \frac{1}{2} \hat{u}_1 \\ &= 0 \end{aligned}$$

References

- Quaas, R. L. 1986. Personal Communication. Animal Science 720. Cornell University, Ithaca, New York.
- Quaas, R. L., Anderson, R. D. and A. R. Gilmour. 1984. Use of Mixed Models for Prediction and for Estimation of (Co) Variance Components. Animal Genetics and Breeding Unit, University of New England, N.S.W., 2351, Australia.

APPENDIX

Summary of predictions of the BV of animals 1 to 15 using RAM, SDM, SMM and SM

| Animal | Prediction of BV | | | |
|--------|------------------|---------|---------|---------|
| | RAM | SDM | SMM | SM |
| 1 | -0.4410 | -0.0674 | 0.0150 | 0.0 |
| 2 | -2.0758 | -0.2030 | 0.0075 | 0.0 |
| 3 | -2.1405 | -0.2708 | 0.0526 | 0.0760 |
| 4 | -1.5212 | -1.3670 | -1.4208 | -1.0624 |
| 5 | 3.5970 | 1.5700 | 1.3908 | 0.9864 |
| 6 | -3.5008 | -0.3158 | 0.1451 | 0.1013 |
| 7 | -3.3839 | -0.5144 | -0.1610 | -0.2599 |
| 8 | -2.4499 | -1.2406 | -1.4764 | -1.1566 |
| 9 | 0.9258 | 0.4706 | 0.7820 | 0.1587 |
| 10 | 3.3714 | 1.5454 | 1.5311 | 1.1566 |
| 11 | -2.8061 | -0.1964 | 0.2326 | 0.1900 |
| 12 | -2.8113 | -1.1230 | -0.6951 | -0.9210 |
| 13 | -2.1962 | -2.3403 | -2.5976 | -2.2661 |
| 14 | 1.8570 | 1.1148 | 1.2018 | 0.7311 |
| 15 | 4.9514 | 2.5082 | 2.5752 | 2.2281 |

Ranking of animals 1 to 15 based on predictions obtained using RAM, SDM, SMM and SM

| Animal | Ranking | | | |
|--------|---------|-----|-----|----|
| | RAM | SDM | SMM | SM |
| 1 | 6 | 6 | 9 | 9 |
| 2 | 8 | 8 | 10 | 9 |
| 3 | 9 | 9 | 8 | 8 |
| 4 | 7 | 14 | 13 | 12 |
| 5 | 2 | 2 | 3 | 3 |
| 6 | 15 | 10 | 7 | 7 |
| 7 | 14 | 11 | 11 | 10 |
| 8 | 11 | 13 | 14 | 13 |
| 9 | 5 | 5 | 5 | 6 |
| 10 | 3 | 3 | 2 | 2 |
| 11 | 12 | 7 | 6 | 5 |
| 12 | 13 | 12 | 12 | 11 |
| 13 | 10 | 15 | 15 | 14 |
| 14 | 4 | 4 | 4 | 4 |
| 15 | 1 | 1 | 1 | 1 |