Lactation Curves and Prediction of Daily and Accumulated Milk Yields in a Multibreed Dairy Herd in Thailand Using All Daily Records[1/]

Skorn Koonawootrittriron[*], Mauricio A. Elzo[2/], Sornthep Tumwasorn, and Wirot Sintala[3/]

Department of Animal Science, Faculty of Agriculture,
Kasetsart University, Bangkok 10900, Thailand.

Abstract

Daily milk records of 88 cows in a multibreed dairy herd were used to describe lactation curves and test the ability of seven models to predict daily milk yields, and accumulated 100-d and 305-d milk yields. The seven mathematical models were a gamma function, a mixed log second-degree polynomial model, and five polynomial regression models. Cows were of three breed groups: Holstein Friesian (HF), 1/2HF-1/2RS (Red Sindhi), and 3/4HF-1/4RS. Lactation number was classified as first, second, third, and fourth-and-later lactations. Calving age was defined as age less than 30 months, and equal to or greater than 30 months for the first lactation; age less than 44 months, and equal to or greater than 44 months for the second lactation; less than 60 months, and equal to or greater than 60 months for the third lactation; and all ages for the fourth-and-later lactations. Seasons of calving were defined as winter (November to February), summer (March to June), and rainy (July to October). Four general types of lactation curves were found: convex, slightly convex, two-peaked, and flat. Types of lactation curves varied across breed group x lactation x calving season and breed group x lactation x calving age subclasses. The second-degree polynomial model was the best in terms of predicted minus actual daily and 305-d milk yields, and computational requirements. For 100-d milk yield the best model was the sixth degree polynomial model. The application of these results is limited to HF x RS multibreed herds in the Northeastern region of Thailand. To further help national genetic evaluation efforts, this study needs to be repeated with a data set that is representative of the national Thai dairy cattle population.

Key words: dairy cattle, lactation curve, milk yield, prediction, multibreed

Introduction

Knowledge of lactation curves and yields in dairy cattle is important for decisions on herd management and selection. Different mathematical models have been evaluated for their ability to describe lactation patterns of milk yield as well as the ability to predict cumulative milk yields from partial records (Schaeffer *et al.*, 1977, Batra, 1986, Vargas *et al.*, 2000). A popular model that has been widely used to describe lactation curves and predicted lactation yields is the gamma function (Wood, 1967). However, a linear regression model of yields on days in lactation (linear and quadratic) and on log of 305 divided by days in lactation (linear and quadratic) was

---

[1/] This research was supported by the Florida Agricultural Experiment Station and a grant from the Thailand Research Fund under the Royal Golden Jubilee Project, and approved for publication as Journal Series No. R-08042.
[2/] Department of Animal Sciences, University of Florida, Gainesville, FL 32611-0910, USA
[*] Correspondence: Present address [3/] Sakon Nakhon Agricultural Research and Training Center, P.O. Box 3, Pungkone, Sakon Nakhon 47160, Thailand. E-mail: skornk@hotmail.com

reported to be better than Wood's gamma function for predicting lactation yields (Ali and Schaeffer, 1987). Recently, more complicated nonlinear models (Grossman *et al.*, 1986, 1999, Morant and Gnanasakthy, 1989) to describe lactation curves have been proposed, but they require additional mathematical techniques and may have more computational problems because of the larger number of parameters that need to be estimated and the greater amount of data is required to estimate those parameters with a reasonable degree of accuracy.

In Thailand, although Holstein Friesian is the main breed used for crossbreeding in dairy herds, most farmers simultaneously raise many types of crossbred dairy cattle in their farms. Each one of these breed groups of cattle can potentially have lactation curves of different shapes depending on calving age and calving season. Because interactions among these effects can be important, if these animals are to participate in a genetic evaluation, then lactation curves within breed group x lactation number x calving age subclasses and breed group x lactation number x calving season subclasses need to be compared. In addition, it is also important to assess the ability of various mathematical models to estimate daily milk yields using all available daily records.

Thus, the objectives of this study were: 1) to compute actual lactation curves based on means of daily milk records, 2) to compare the ability of seven mathematical models including a gamma function, a regression model and five polynomial regression models to describe lactation curves of individual cows, and 3) to compare the ability of these seven models to predict cumulative 100-d and 305-d milk yields of individual cows, within breed group x lactation number x calving age and breed group x lactation number x calving season subclasses, using *all daily milk yields* in a multibreed dairy herd in Thailand.

Materials and Methods

Animals, Management, and Records

Analyses were performed on data provided by the Sakon Nakhon Agricultural Research and Training Center (SARTC) and collected in an experimental multibreed dairy herd in northeast Thailand between November 1, 1997 and December 31, 1999. Animals consisted of purebred Holstein Friesian (HF) and crossbred Holstein Friesian x Red Sindhi (HF-RS), which were under the cattle breeding project of Rajamangala Institute of Technology (RIT).

A total of 28,452 daily milk lactation records (days 5 to 305) from 106 lactations of 88 straightbred HF and crossbred HF x RS cows were used here (Table 1). There were three breed groups of cows: HF, 1/2HF-1/2RS, and 3/4HF-1/4RS. Lactation records were from first, second, third, and fourth-and-later lactations.

Calving seasons were classified as winter (November to February), summer (March to June), and rainy (July to October). Because of the overlapping of calving ages and lactation numbers, calving age x lactation number subclasses were defined. The resulting seven calving age x lactation subclasses were: 1) calving age less than 30 months x lactation 1, 2) calving age equal to or greater than 30 months x lactation 1, 3) calving age less than 44 months x lactation 2, 4) calving age equal to or greater than 44 months x lactation 2, 5) calving age less than 60 months x lactation 3, 6) calving age equal to or greater than 60 months for the third lactation, and 7) calving age greater than 60 months x lactation 4 and greater.

Animals of all breed compositions were raised under the same nutritional and management conditions. All cows were milked twice a day, once in the morning (0400 hrs.) and once in afternoon (1400 hrs.). Irrigated Napier (*Penisetum pupureum*) and Guinea (*Panicum maximum*) grasses were fresh cut and carried to feed all animals. However, the quantity and quality of these green chops was variable and changed from season to season depending on the weather in a particular year. During the dry season, milking cows were maintained on Ruzi (*Brachiaria ruziziensis*) pastures after the morning milking. Mineral blocks were provided as supplement *ad libitum*. Animals were fed 8 kg of a concentrate made at the research station using local grains, crop-residues, modified-rice straw, molasses, vitamins, and minerals.

All animals in the herd were treated against internal (IVOMEC®, ABENDAZOLE®) and external parasites (BARICATE®) every six months. Breeding age and younger heifers were vaccinated against viral (e.g., *Foot and Mouth Disease, Hemorrhagic Disease*) and bacterial (e.g., *Hemophilus sp, Leptospirosis sp*) diseases annually beginning at six months of age. The parasite control and vaccination program used in the HF-RS multibreed herd followed the guidelines given by the Department of Livestock Development of Thailand (SARTC, 1999).

Cows were mated throughout the year to maintain a minimum level of total amount of milk produced per day. Qualified personnel observed signs of estrous behavior daily. Animals were inseminated up to three times. Cows were palpated by a veterinarian 60 d after the last insemination. Open cows were placed with a Holstein or a Red Sindhi clean up bull according to the mating plan. There were 2 Holstein Friesian and 2 Red Sindhi clean up bulls available every year. Pregnant cows were dried off two months before calving.

Table 1. General description of the data set

	HF	1/2HF-1/2RS	3/4HF-1/4RS	Total
Cows	75	8	5	88
Lactations	88	13	5	106
First lactations	18	4	5	27
Second lactations	43	6	-	49
Third lactations	9	3	-	12
Fourth and more lactations	18	-	-	18
Number of observation (daily yields)	23,986	2,972	1,494	28,452

Models and Data Analysis

Genetic evaluation models require the construction of contemporary groups where comparisons among predicted genetic values of animals for traits of economic importance can be made in a fair manner. For this Thai data set, one such contemporary group is the subclass formed by cows from the same breed group x calving age group x lactation number x calving season subclass. Unfortunately, preliminary analyses showed that it was unfeasible to use these subclasses because 56% of them were either empty or had a single lactation. Consequently, two larger subclasses were defined: 1) breed group x lactation x calving season, and 2) breed group x lactation x calving age. The data set yielded 20 breed group x lactation x calving season subclasses, and 13 breed group x lactation x calving age subclasses. Analyses were conducted separately for each one of the breed group x lactation x calving season and the breed group x lactation x calving age subclasses.

Lactation curves were constructed using means of individual cow daily milk yields (5 to 305d) within each subclass. Means of daily milk yields within breed group x lactation x calving season and breed group x lactation x calving age subclass were plotted using Microsoft Excel 2000 (Dodge and Stinson, 1999).

Daily milk yields (5 to 305 d) for individual cows were predicted using each of the seven equations used to model lactation curves within each subclass. The predictive ability of these seven equations was tested using deviations of predicted minus actual daily milk yields within lactations. Least squares means of daily deviations of predicted minus actual daily productions were computed for each breed group x lactation x calving season and breed group x lactation x calving age subclass. To evaluate the predictive ability of the seven models, least squares means of differences between predicted and actual daily milk yields were tested (t-test) for their difference with respect to zero, and for differences between lactation models. The statistical model used was:

$$d_{ijkl} = \mu + subclass_i + model_j + cow_k + e_{ijkl}$$

where

d_{ijkl}	=	difference between predicted and actual milk production in lactation day j of cow k within subclass i and model j,
μ	=	overall mean,
$subclass_i$	=	i^{th} breed group x lactation x calving season or breed group x lactation x calving age,
$model_j$	=	j^{th} prediction model,
cow_k	=	k^{th} cow,
e_{ijkl}	=	residual.

All effects in the model were assumed to be fixed, except for the residual term that was assumed to be independent, identically distributed with mean zero and a common variance. Because a cow will be in each subclass type, separate computations were done for breed group x lactation x calving season and breed group x lactation x calving age subclasses.

Individual cow actual and predicted 100-d and 305-d milk yields were compared to evaluate the performance of the seven models to predict milk yield for two traits of economic importance used in genetic evaluation of dairy cattle. Least squares means of 100-d and 305-d differences between predicted and actual milk yields were also computed for each breed group x lactation x calving season and breed group x lactation x calving age subclass for comparison purposes. These subclass deviations were used to assess (t-tests) the ability of the seven models to predict 100-d and 305-d milk yields. The statistical model used was:

$$d_{ijk} = \mu + subclass_i + model_j + e_{ijk}$$

where

d_{ijk}	=	difference between predicted and actual milk production of cow k at 100-d or at 305-d of lactation, within subclass i and model j,
μ	=	overall mean,
$subclass_i$	=	i^{th} breed group x lactation x calving season or breed group x lactation x calving age,
$model_j$	=	j^{th} prediction model,
e_{ijkl}	=	residual.

All effects in the model were assumed to be fixed, except for the residual term that was assumed to be independent, identically distributed with mean zero and a common variance. Separate computations were done for breed group x lactation x calving season and breed group x lactation x calving age subclasses.

Models that had smaller differences between predicted and actual milk yields within breed group x lactation x calving season and breed group x lactation x calving age subclasses were considered to have better predictive ability.

The seven mathematical models used here were as follows.

Model 1: Wood gamma function (Wood, 1967),

$$y_t = at^b e^{-ct} \quad [1]$$

where y_t is the milk yield on day *t* in each subclass, ***a*** is the initial yield of lactation, ***b*** represents the increasing slope, and ***c*** represents the decreasing slope. Because nonlinear regression does not guarantee convergence (SAS, 1990), natural logarithms were taken on both sides of equation [1] giving

$$\ln y_t = \ln a + b \ln t - ct + \varepsilon_t$$

where e_t is the residual. The predicted yield on day *t* (y_t) was computed as $\mathbf{y_t} = exp\ (ln\ y_t)$.

Model 2: mixed log second-degree polynomial model of milk yie ld on day in lactation and log of day in lactation (linear and quadratic), of Ali and Schaeffer (1987),

$$y_t = b_0 + b_1\gamma_t + b_2\gamma_t^2 + b_3 w_t + b_4 w_t^2 + e_t \quad [2]$$

where y_t is the milk yield on day t in each subclass, $\gamma_t = t/305$, $w_t = \ln(305/t)$, t = days since calving or days in milk, b_0, b_1, b_2, b_3, and b_4 are the regression coefficients where b_0 is associated with peak yield, b_3 and b_4 the increasing slope of the curve and b_1 and b_2 are associated with the decreasing slope; and e_t is the residual.

Models 3, 4, 5, 6 and 7: second, third, fourth, fifth, and sixth polynomial regression models, respectively,

Model 3: $y_t = b_0 + b_1 t + b_2 t^2 + e_t$ [4]

Model 4: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + e_t$ [5]

Model 5: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + e_t$ [6]

Model 6: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + e_t$ [7]

Model 7: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + b_6 t^6 + e_t$ [8]

where y_t is the milk yield on day t in each subclass, t = days since calving or days in milk, b_0, b_1, b_2, b_3, b_4, b_5 and b_6 are coefficients, and e_t is the residual.

Estimates of regression coefficients for models 1 through 7 were obtained using PROC REG of the SAS program (SAS, 1990). This program was also used to compute predicted daily milk yields (5 to 305 d) for all models.

Individual cow actual milk yields at 100-d and 305-d, predicted minus actual milk yield daily deviations, and predicted 100-d, and 305-d milk yield deviations were computed using the general SAS program (SAS, 1990). Least squares means of milk yield deviations per subclass were obtained using LSMEANS statement of PROC GLM (SAS, 1990).

Predicted means of daily milk yields per breed group x lactation x calving season and breed group x lactation x calving age subclass using the seven models above were drawn using Microsoft Excel 2000 (Dodge and Stinson, 1999). These lines were compared with the corresponding lines of daily milk yields within subclasses.

Results and Discussion

Actual Lactation Curves

Plots of actual lactation curves showed that the shape of actual lactation curves varied across breed group x lactation x calving season and breed group x lactation x calving age subclasses. Lactation curves were classified into four types according to the pattern of milk production from day 5 to 305. Type 1: milk production increased continuously from the beginning of lactation, it reached a peak between 50 to 70 d of lactation, then it decreased until the end of the lactation (convex line); type 2: milk production peaked before 30 d of lactation, then it steadily decreases to the end of lactation (slightly convex line); type 3: milk production peaked twice, one between 30 to 70 d of lactation, and another between 180 and 210 d of lactation (double peak line); and type 4: milk production remained fairly constant throughout the lactation (flat line).

Breed group subclasses. First lactation curves of HF cows were of type 2, their milk production increased after calving to reach a small peak around 30 d of lactation, and then it decreased until the end of their lactations. Crossbred 1/2HF-1/2RS cows had flat lactation curves (type 4), and 3/4HF-1/4RS cows had type 1 lactation curves, with a peak in milk production at about 75 d of lactation.

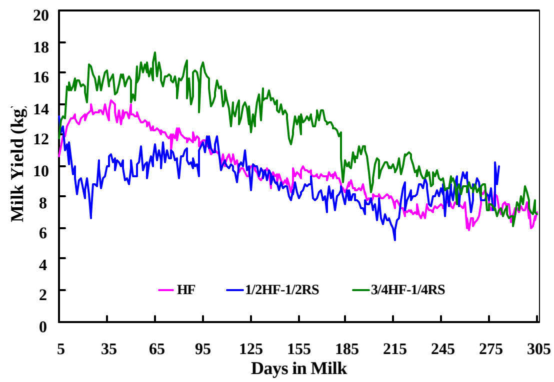

Figure 1. Means of first lactation curves of HF, 1/2HF-1/2RS, and 3/4HF-1/4RS cows that calved at ages equal to or greater than 30 mo

Figure 1 shows means of first lactation curves of HF, 1/2HF-1/2RS, and 3/4HF-1/4RS cows that calved at ages equal to or greater than 30 months. The mean peak milk yield of HF cows (14.2 kg at 37 d in milk) was lower than that of 3/4HF-1/4RS cows (17.3 kg at 65 d in milk), but higher than the one from 1/2HF1/2RS cows (13.2 kg at 5 d in milk). The shape of the mean of lactation curves of 1/2HF1/2RS cows was flatter than those of HF and 3/4HF-1/4RS in this calving age subclass.

Tekerli *et al.* (2000) indicated that Holstein cows with flat lactation curves would be expected to show higher persistency and higher milk yields per lactation. However, when comparing cows across breed groups this may not be the case. Here (Figure 1), the pattern mean lactation curves of 1/2HF-1/2RS cows that calved at ages equal to or greater than 30 months was flatter than that of HF and 3/4HF-1/4RS cows, however, they had lower total 305-d milk yields. The means of 305-d milk yields of first lactations of HF, 1/2HF-1/2RS, and 3/4HF-1/4RS cows that calved at ages equal to or greater than 30 months were 2,723.6, 2,014.5 and 3,579.0 kg, respectively. Milk production in these three groups was clearly related to the fraction of Holstein genes. Higher fractions (HF, and 3/4HF-1/4RS) produced more milk than 1/2HF-1/2RS. The 3/4HF-1/4RS group of cows produced more milk than purebred Holstein cows probably due to the inability of Holstein cows to cope with the environmental conditions (heat, humidity, internal and external parasite load) at the SARTC farm in Thailand.

The results obtained here reconfirmed those obtained in previous studies conducted at SARTC. SARTC (1999) reported that HF raised under Northeastern conditions of Thailand needed more veterinary care than HF x RS crossbred cows because of environmental stresses and tropical diseases. The F1 (1/2HF-1/2RS) cows were healthier, but had lower milk production than HF and 3/4HF-1/4RS. Crossbred 3/4HF-1/4RS and higher HF fraction (up to 87.25 %HF) cows produced more milk than 1/2HF-1/2RS and HF cows (SARTC, 1999).

Later lactations in HF cows had similar patterns of lactation curves to the first lactation, except that they had higher peak yields. On the other hand, the shape patterns of later lactation curves of 1/2HF-1/2RS cows were different from those of their first lactations. Later lactations of 1/2HF-1/2RS cows were of type 2 (slightly convex) instead of type 4 (flat). In all breed groups initial and peak milk yield increased from lactation 1 to 3.

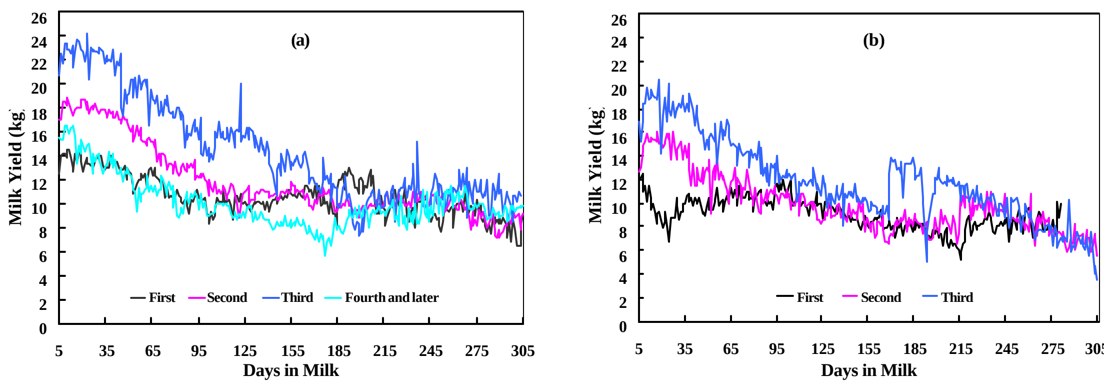

Figure 2. Means of lactation curves of various lactation numbers in HF cows that calved in summer (a), and in 1/2HF-1/2RS cows that calved in winter (b)

Figures 2a and 2b contain examples of the types of lactation curves found in the HF-RS herd of this study. Figure 2a shows that the means of first lactation curves of HF cows that calved in summer had lower initial (12.5 kg) milk productions than those of second (17.1 kg), third (20.8 kg), and fourth-and-later (15.5 kg) lactations. All these lactations were of type 2; they had a short initial increase in milk production, reached a peak yield within the first month of lactation, and then decreased at various rates depending on the lactation number until 305 d. The mean peak yield of first, second, third, and fourth-and-later lactations cows were 14.6 kg (at 10 days in milk), 18.96 kg (at 10 days in milk), 24.2 kg (at 23 days in milk), and 16.6 kg (at 8 days in milk), respectively.

The means of first lactation curves of 1/2HF-1/2RS cows that calved in winter (Figure 2b) were similar to those of HF cows. They had an initial milk yield (12.2 kg) that was lower than that observed for second (12.7 kg) and third lactations (16.9 kg). The mean peak yield in the first lactation of 1/2HF-1/2RS cows (12.5 kg at 7 days in milk) was also lower than that in the second (16.0 kg at 17 days in milk) and in the third lactation (20.4 kg at 18 days in milk). Comparison of first and later lactations could not be done with 3/4HF-1/4RS cows because they had only first lactation records.

Mean peak yields were lower and were reached earlier in the first lactation because cows needed to devote a substantial portion of their nutritional intake to their growth and development. Because 1/2HF-1/2RS mature earlier than HF cows (4 years *vs.* 5 years; SARTC, 1999), nutritional requirements allocated to growth and development during the second lactation were probably lower for 1/2HF-1/2RS than for HF cows. Thus, 1/2HF-1/2RS probably utilized more nutrients for milk production than HF cows, resulting in longer peak yield times for 1/2HF-1/2RS animals. The degree of maturity of 1/2HF-1/2RS and HF cows was probably similar at the third lactation, thus mean peak yields were achieved at similar days. Mean peak yield would decrease as cows get older and less productive (SARTC, 1999), as found here for fourth-and-later-lactation HF cows.

Calving age subclasses. The patterns of shape of lactation curves of older calving ages within lactation number were similar to those of the younger calving ages within lactations. Older cows of all breed groups had type 2 lactation curves; milk production increased after calving for a short time, reached a peak in less than a month, and then steadily decreased until 305 d.

Plots of means of first lactation curves in Figure 3 show that HF cows that calved at less than 30 months of age (12.3 kg at 51 days in milk) had peak yields slightly lower than those of cows calved at ages equal to or greater than 30 months (13.4 kg at 25 days in milk). The difference in peak yield between younger and older cows in the first lactation (1.1 kg) was smaller than that in the second lactation (4.4 kg), and in the third lactation (5.3 kg). In addition, younger cows had flatter lactation curves than older cows in all lactations. In contrast, 3/4HF-1/4RS cows calving at less than 30 months of age had peak yields (17.0 kg at 75 days in milk) higher than those of cows that calved at ages equal to or greater than 30 months (16.1 kg at 65 days in milk). Differences in daily milk production and peak yields translated in an advantage of approximately 200 kg milk for later lactations over first lactations in HF cows, and of about 100 kg in 3/4HF 1/4RS cows. These small differences suggest that it might be advantageous to breed cows to calve at younger ages within lactations, provided adequate feed is available. An additional advantage would be a potential longer herd life of younger calving cows.

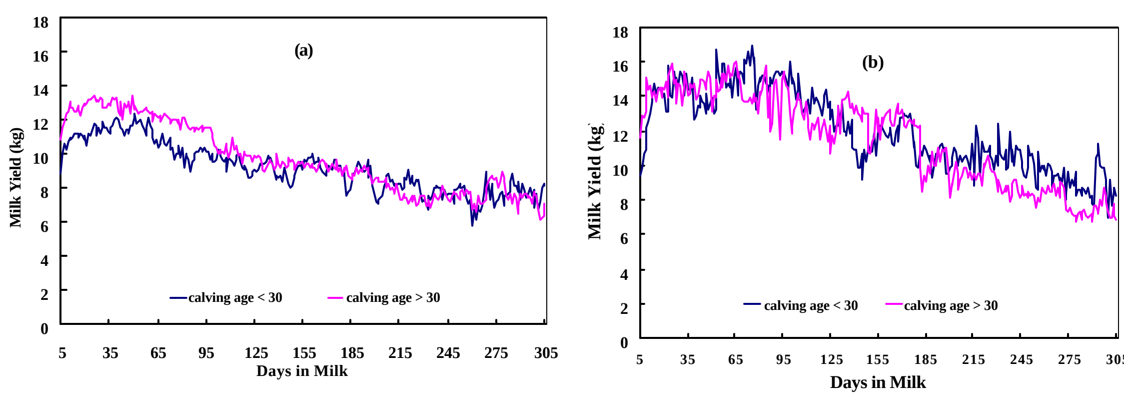

Figure 3. Means of first lactation curves of HF (a) and 3/4HF-1/4RS (b) cows that calved at less than 30 months, and equal to or greater than 30 months of age

Calving season subclasses. Cows calving in different seasons had different types of lactation curves. The HF cows that calved in winter had lactation curves of type 2 (convex lactation curve), while cows that calved in summer were of type 3 (two peaks per lactation), and cows that calved in the rainy season had lactations of type 4 (flat lactation curve). Examples of the types of lactation curves by calving season are shown in Figure 4. The initial yield of lactation of HF cows calved in the rainy season (7.5 kg) was lower than that of HF cows calved in winter (10.6 kg), and summer (12.5kg). The peak yield of the first lactation of HF cows that calved in the rainy season (11.5 kg) was lower than that of cows that calved in winter (14.2 kg), and summer (14.6 kg).

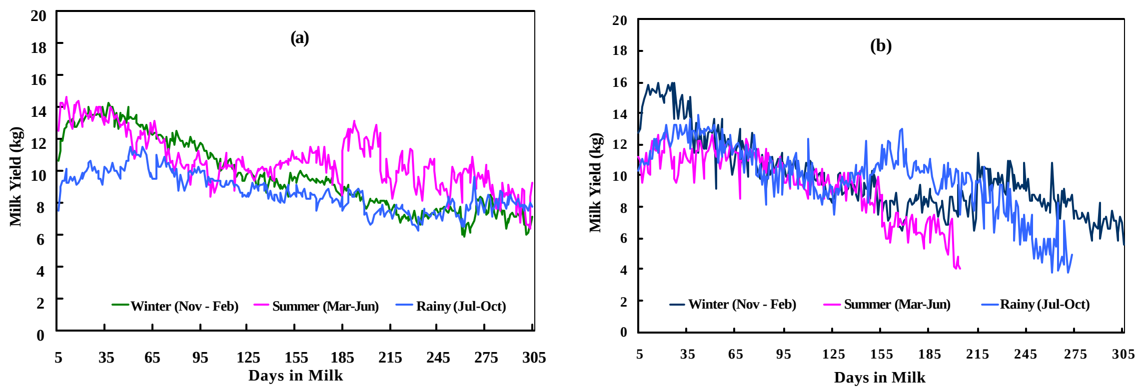

Figure 4. Means of lactation curves of first lactation HF (a) and second lactation 1/2HF-1/2RS (b) cows that calved in winter, summer, and in the rainy season

Holstein Friesian cows that calved in the rainy season had type 4 (flat) lactation curves with lower initial production and peak yields than cows that calved in winter and summer. The lower production of HF cows in the rainy season was likely due to the incidence of mastitis, laminitis, and milk fever problems, which was much greater than in winter and summer. SARTC (1999) also found a similar pattern of health problems for HF cows. Perhaps a seasonal breeding strategy that prevents HF cows from calving in the rainy season could be implemented. On the other hand, 1/2HF 1/2RS cows had no laminitis and milk fever problems. Differences in frequency of health problems across seasons cannot be evaluated here for 1/2HF 1/2RS, however, because the number of 1/2HF 1/2RS cows per season was very small (1 to 2).

Lactation curves of cows that calved in summer had two peaks per lactation (type 3); the first one occurred during early part of lactation (60 d) and second one occurred during the late part of the lactation (210 d). The second peak of summer calving HF cows was probably related to the high quantity and quality of grass available in the rainy season. Wood (1972) suggested that the rate of descent of the slope of lactations was dependent on seasonal availability of grass. Thus, if the quality and quantity of roughage between the two lactation peaks of cows that calve in the summer could be increased, then the shape of the summer lactation curves could be changed to a type 1 (single peak at 60 d), and this way milk production per lactation could be increased.

Initial milk yields, peak yields, and rates of decrease of daily milk yields across calving seasons in this study differed from those reported in other countries. In Brazil, Madalena *et al.* (1979) reported that HF and HF-Gir cows that calved in the rainy season had higher initial milk yields than cows that calved in the dry season, which had more persistent lactations. In Turkey, Tekerli *et al.* (2000) reported that Holstein cows that calved during the rainy season and in winter reached higher peak and lactation yields than those that calved in the dry season. These across country differences are likely to be due primarily to environmental effects. Also, these results stress the fact that seasonal effects affect the shape of lactation curves.

Predicted daily yields

Calving age subclasses. Least squares (LS) means of (predicted minus actual) daily milk yield deviations of the seven models by breed group x lactation number x calving age subclasses are presented in Table 2. These LS means indicated that all models had similar patterns of daily milk yield predictive abilities for first-lactation HF cows across calving ages. All models predicted daily milk yields accurately (predicted minus actual daily milk yield deviations close to zero, and nonsignificant), except for model 1 (gamma function) which gave significant differences for all calving ages ($P < 0.01$). For the first lactation of 1/2HF-1/2 RS cows, model 1 (gamma function) was the only model that gave significant differences between mean predicted and actual daily milk yields ($P < 0.05$). Similarly, mean differences for first lactations 3/4HF-1/4RS cows were significant for one out of two calving ages. Other subclasses gave nonsignificant differences for model 1.

On the other hand, models 2 to 7 yielded small and nonsignificant mean predicted minus actual daily milk yield differences for cows of all breed groups, lactation numbers, and calving ages (Table 2). Thus, any one of models 2 through 7 could be used to predict daily milk yields with reasonable accuracy. However, based on the actual LS means of the predicted minus actual daily milk yield differences, models 2 (mixed log second degree polynomial) and 3 (second degree polynomial) gave similar high accuracies, followed by model 5 (fourth degree polynomial).

Table 2. Least squares means of the differences between predicted and actual daily milk yields by breed group x lactation number x calving age subclasses

Breed group[1/]	Lactation number	Calving age	No. of lactations	Actual yields (kg)	Models[2/] 1	2	3	4	5	6	7
HF	1	< 30 mo	8	9.4	-0.1182**	0.0007	0.0001	0.0008	0.0006	-0.0003	-0.0002
HF	1	$\geq$ 30 mo	10	9.9	-0.0976**	0.0016	0.0004	0.0004	0.0014	-0.0002	-0.0003
HF	2	< 44 mo	19	10.2	-0.1242**	-0.0005	0.0001	-0.0003	0.0001	-0.0007	-0.0003
HF	2	$\geq$ 44 mo	24	12.0	-0.1420**	0.0003	-0.0001	0.0005	0.0002	-0.0001	0.0005
HF	3	< 60 mo	6	11.0	-0.0811**	-0.0002	0.0001	-0.0007	0.0010	-0.0001	-0.0002
HF	3	$\geq$ 60 mo	3	12.2	-0.1553*	0.0002	-0.0006	-0.0009	-0.0005	-0.0027	-0.0024
HF	$\geq$4	all ages	18	10.9	-0.1470**	-0.0009	0.0000	0.0004	-0.0006	-0.0008	-0.0001
1/2HF 1/2RS	1	$\geq$ 30 mo	4	9.2	-0.0977*	-0.0009	0.0007	-0.0028	-0.0012	0.0003	-0.0028
1/2HF 1/2RS	2	< 44 mo	3	10.6	-0.1243*	0.0019	-0.0001	-0.0009	-0.0001	0.0003	-0.0019
1/2HF 1/2RS	2	$\geq$ 44 mo	3	9.5	-0.0772	0.0011	0.0014	0.0011	-0.0003	0.0016	0.0016
1/2HF 1/2RS	3	< 60 mo	3	11.9	-0.0662	0.0010	0.0022	-0.0025	0.0012	0.0018	0.0006
3/4HF 1/4RS	1	< 30 mo	2	12.0	-0.1029	0.0014	-0.0011	-0.0026	-0.0004	-0.0002	-0.0017
3/4HF 1/4RS	1	$\geq$ 30 mo	3	11.6	-0.1228*	-0.0003	0.0010	0.0000	-0.0006	-0.0012	-0.0003
All	All	All	106	10.86	-0.1236*	-0.0000	-0.0000	-0.0002	0.0000	-0.0004	-0.0003

[1/] HF = Holstein Friesian, RS = Red Sindhi

[2/] Model 1: $y_t = at^b e^{-ct}$ Model 2: $y_t = b_0 + b_1\gamma_t + b_1\gamma_t^2 + b_3 w_t + b_4 w_t^2 + e_t$

Model 3: $y_t = b_0 + b_1 t + b_2 t^2 + e_t$ Model 4: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + e_t$

Model 5: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + e_t$ Model 6: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + e_t$

Model 7: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + b_6 t^6 + e_t$

Where y_t is milk yield at day in lactation t, t = days in lactation, $\gamma_t = t/305$, $w_t = \ln(305/t)$

* significant ($P < 0.05$) for Ho: LSMEAN=0, ** highly significant ($P < 0.01$) for Ho: LSMEAN=0.

Calving season subclasses. The pattern of predicted minus actual daily yields in breed group x lactation number x calving season subclasses was identical to that of breed group x lactation number x calving age subclasses. Model 1 had significant (at least $P < 0.05$) predicted minus actual daily milk yield differences for all subclasses that had 3 or more observations, whereas models 2 through 7 had nonsignificant differences for all subclasses. Models 2 and 3 were again the ones that gave the smallest predicted minus actual daily yield mean differences.

The overall ranking of the seven lactation prediction models, considering both sets of subclasses, was as follows: 1) models 2 (mixed log second degree polynomial) and 3 (second degree polynomial), 2) model 5 (fourth degree polynomial), 3) model 4 (third degree polynomial), 4) model 7 (sixth degree polynomial), 5) model 6 (fifth degree polynomial), and 6) model 1 (gamma function).

Predicted lactation curves. Predicted lactation curves of individual cows computed using models 2 through 7 followed their actual lactation curves more closely than that of model 1. Predicted lactation lines of models 2 to 7 yielded smaller predicted minus actual daily yield differences than model 1. This allowed the predicted lines from models 2 through 7 to closely mimic the jagged shape of the actual lactation curves of individual cows. The predicted lactation line from model 1 was smoother than those of models 2 to 7, and had substantially larger predicted minus actual daily milk yield differences.

As an illustration of the behavior of the seven prediction models studied here, Figures 5, 6, and 7 show plots of means of actual and predicted individual cow lactation curves using models 1 through 7 for the subclass HF x lactation 1 x calving age less than 30 months. Plots are ordered by model number from 1 to 7. Model 1 is noticeably worse than models 2 through 7 to fit the lactation curves of individual cows in this subclass. Furthermore, model 1 underestimated predicted daily milk yields from day 5 to 65 and slightly overestimated daily milk yields from day 155 to 215 of lactation. Rowlands *et al.* (1982) found that in British Friesian the gamma function (model 1) also underestimated daily milk yield between calving and 84 d of lactation, and overestimated daily milk yield from day 85 to 175 of lactation. These results were different from Cobby and Le Du (1978) who reported that model 1 overestimated daily milk yield after calving up to 70 d in lactation and underestimated from 70 to 140 d in lactation. Discrepancies between these studies may be due to differences in time to peak yield, differences in breed composition, and seasonal effects.

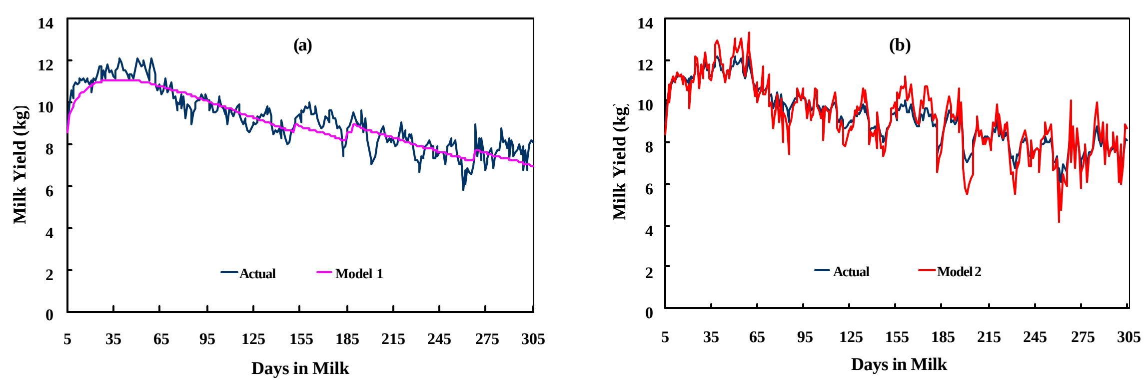

Figure 5. Means of actual and predicted individual cow lactation curves using models 1 (a) and 2 (b) for the subclass HF x lactation 1 x calving age less than 30 months

Plots 2 to 7 in Figure 5, 6 and 7 show how closely models 2 to 7 fit lactation lines of individual cows. Except for a slight overestimation of individual milk yields during the first 65 d of lactation, these models show how well they estimated daily milk production of individual cows. Any one of these models (2 to 7) would seem appropriate for use in genetic evaluation procedures. However, considering the degree of complexity of models 2 to 7, then model 3 would seem the best compromise between daily milk yield predictive ability and computational simplicity.

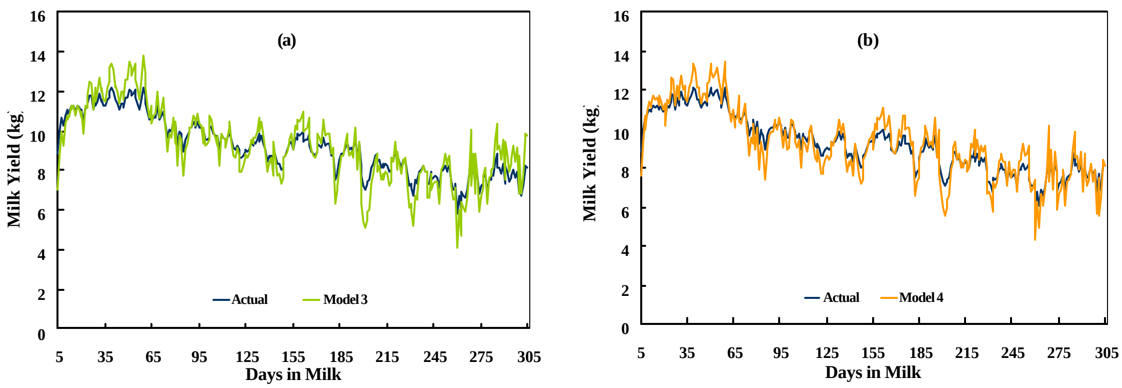

Figure 6. Means of actual and predicted individual cow lactation curves using models 3 (a) and 4 (b) for the subclass HF x lactation 1 x calving age less than 30 months

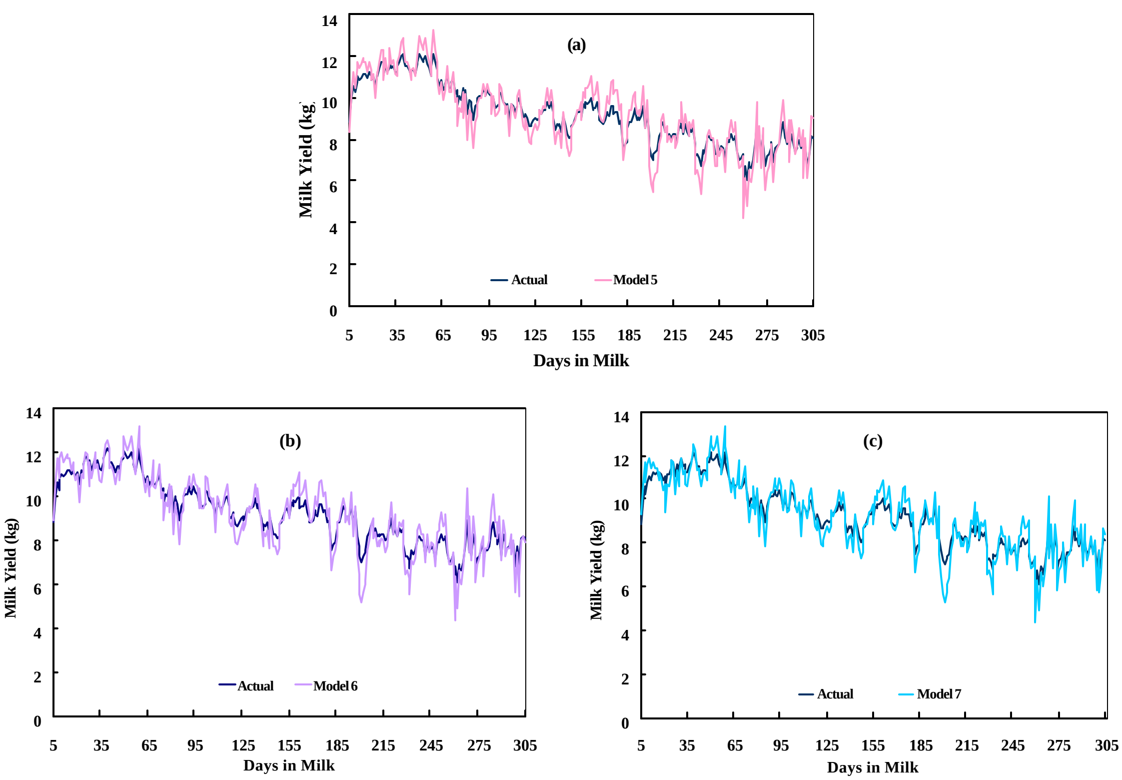

Figure 7. Means of actual and predicted individual cow lactation curves using models 5 (a), 6 (b) and 7 (b) for the subclass HF x lactation 1 x calving age less than 30 months

Prediction of 100-d milk yields

Calving age subclasses. Least squares means of 100-d milk yield deviations of the seven models by breed group x lactation number x calving age are presented in Table 3. Models 5 through 7 predicted 100-d milk yields accurately (predicted minus actual 100-d milk yield deviation close to zero, and non significant) for all subclasses. All but one ($P < 0.05$) of the LS means differences for models 2 and 4 were nonsignificant. The least accurate models were models 1 and 3; model 1 had six significant (at least $P < 0.05$) differences, and model 3 had four significant differences (at least $P < 0.05$). All significant differences were due to underestimation of daily milk production around the peak yield of cows in these subclasses. Most cows in these subclasses had lactation type 2 (slightly convex line). Underestimation of daily milk yields affected primarily the prediction of 100-d yields of first lactation HF cows, second lactation HF cows younger than 44 months at calving, third lactation HF cows younger than 60 months at calving, fourth-and-later lactation HF cows, and first lactation 1/2HF 1/2RS cows.

Table 3. Least squares means of the differences between predicted and actual 100-d milk yields by breed group x lactation number x calving age subclasses

Breed group[1/]	Lactation Number	Calving age	No. of lactations	Actual yields (kg)	Models[2/]						
					1	2	3	4	5	6	7
HF	1	< 30 mo	8	1,061.5	-15.6*	-2.7	7.8	3.3	-1.1	-0.5	1.0
HF	1	≥ 30 mo	10	1,210.9	-39.0**	9.4*	22.8**	15.8**	4.7	2.9	6.9
HF	2	< 44 mo	19	1,072.3	13.3*	-6.3	-2.5	-5.3	-6.1	-6.5	-2.2
HF	2	≥ 44 mo	24	1,394.8	4.4	0.9	10.2*	4.0	-2.2	-2.5	0.1
HF	3	< 60 mo	6	1,214.7	14.6**	-1.1	2.2	1.7	-2.7	-5.8	0.3
HF	3	≥ 60 mo	3	1,401.4	-1.4	-10.0	-0.2	-11.3	-10.1	-6.2	-3.7
HF	≥4	all ages	18	1,261.3	-9.7*	4.9	8.4*	6.3	1.3	0.6	1.0
1/2HF 1/2RS	1	≥ 30 mo	4	900.7	-18.8*	2.5	9.7	3.6	2.5	3.7	2.1
1/2HF 1/2RS	2	< 44 mo	3	1,228.8	9.2	-0.4	-1.1	0.1	-6.0	-6.5	-3.9
1/2HF 1/2RS	2	≥ 44 mo	3	1,125.1	5.9	1.4	4.6	0.0	-2.7	-0.9	1.3
1/2HF 1/2RS	3	< 60 mo	3	1,481.4	14.0	-5.9	3.8	-3.8	-9.6	-7.7	-4.1
3/4HF 1/4RS	1	< 30 mo	2	1,415.9	-30.3	16.6	43.2*	29.6	3.1	2.5	5.4
3/4HF 1/4RS	1	≥ 30 mo	3	1,417.0	10.1	3.9	17.0	10.5	3.0	1.2	8.2
All	All	All	106	1,234.3	-2.5	0.6	8.0**	3.4	-1.7	-2.1	0.7

[1/] HF = Holstein Friesian, RS = Red Sindhi

[2/] Model 1: $y_t = at^b e^{-ct}$ Model 2: $y_t = b_0 + b_1\gamma_t + b_1\gamma_t^2 + b_3 w_t + b_4 w_t^2 + e_t$

Model 3: $y_t = b_0 + b_1 t + b_2 t^2 + e_t$ Model 4: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + e_t$

Model 5: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + e_t$ Model 6: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + e_t$

Model 7: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + b_6 t^6 + e_t$

Where y_t is milk yield at day in lactation t, t = days in lactation, $\gamma_t = t/305$, $w_t = \ln(305/t)$

* significant ($P < 0.05$) for Ho: LSMEAN=0, ** highly significant ($P < 0.01$) for Ho: LSMEAN=0.

Calving season subclasses. There were four models that gave nonsignificant LS means of 100-d milk yield predicted minus actual differences in the set of breed group x lactation number x calving season subclasses: models 2, 5, 6, and 7. Model 4 had only one significant difference ($P < 0.05$), model 3 had 4, and model 1 had 10 significant differences (at least $P < 0.05$). Thus, the predictive ability of these models in breed group x lactation number x calving season subclasses was essentially the same as that in breed group x lactation number x calving age subclasses described above. Again, model 1 was the worst, model 3 was intermediate, and the best fitting models were models 2, 4, 5, 6, and 7. Similarly, the most affected subclasses were those that had HF cows.

The overall ranking of the seven prediction models for 100-d milk yield prediction based on: 1) the number of nonsignificant predicted minus actual differences in both types of subclasses, and 2) their overall LS mean of predicted minus actual differences, was (best to worst): 1) model 7 (sixth degree polynomial), 2) model 5 (fourth degree polynomial), 3) model 6 (fifth degree polynomial), 4) model 2 (mixed log second degree polynomial), 5) model 4 (third degree polynomial), 6) model 3 (second degree polynomial), and 7) model 1 (gamma function).

Prediction of 305-d milk yields

Calving age subclasses. Table 4 presents the LS means of 305-d milk yield deviations (predicted minus actual 305-d milk yield) by breed group x lactation number x calving age subclasses. All models had LS means of 305-d milk yield deviations close to zero and nonsignificant, except for model 1 which gave highly significant differences ($p<0.01$) for all breed group x lactation number x calving age subclasses. Although models 2 to 7 could be used for 305-d milk yield genetic evaluations, model 3 should probably be chosen because it has the smallest computing requirements.

Table 4. Least squares means of the differences between predicted and actual 305-d milk yields by breed group x lactation number x calving age subclasses

Breed group[1/]	Lactation number	Calving age	No. of lactations	Actual yields (kg)	Models [2/]						
					1	2	3	4	5	6	7
HF	1	< 30 mo	8	2,356.9	-29.69**	0.04	-0.10	0.06	0.01	-0.20	-0.19
HF	1	$\geq$ 30 mo	10	2,723.6	-26.89**	0.32	-0.01	0.01	0.28	-0.16	-0.02
HF	2	< 44 mo	19	2,827.1	-34.56**	-0.15	0.03	-0.07	0.04	-0.18	-0.08
HF	2	$\geq$ 44 mo	24	3,442.6	-40.65**	0.04	-0.10	0.07	0.00	-0.04	0.08
HF	3	< 60 mo	6	3,292.9	-24.32**	-0.07	0.02	-0.20	0.30	-0.02	-0.07
HF	3	$\geq$ 60 mo	3	3,398.6	-43.23**	0.23	0.00	-0.07	0.03	-0.57	-0.50
HF	$\geq$4	all ages	18	2,700.4	-36.33**	-0.10	0.12	0.02	-0.03	-0.08	0.08
1/2HF 1/2RS	1	$\geq$ 30 mo	4	2,014.2	-21.08**	0.08	0.43	-0.33	0.03	0.35	-0.33
1/2HF 1/2RS	2	< 44 mo	3	2,566.6	-30.07**	0.37	-0.10	-0.30	-0.07	0.00	-0.53
1/2HF 1/2RS	2	$\geq$ 44 mo	3	2,137.2	-17.53**	0.13	0.20	0.13	-0.03	0.27	0.27
1/2HF 1/2RS	3	< 60 mo	3	2,755.7	-15.53**	0.10	0.37	-0.20	0.13	0.27	0.00
3/4HF 1/4RS	1	< 30 mo	2	3,579.0	-30.65**	0.40	-0.35	-0.80	-0.15	-0.10	-0.55
3/4HF 1/4RS	1	$\geq$ 30 mo	3	3,478.2	-36.83**	-0.13	0.27	-0.03	-0.23	-0.40	-0.13
All	All	All	106	2,915.2	-33.15**	0.02	0.02	-0.04	0.04	-0.09	-0.05

[1/] HF = Holstein Friesian, RS = Red Sindhi

[2/] Model 1: $y_t = at^b e^{-ct}$ Model 2: $y_t = b_0 + b_1\gamma_t + b_1\gamma_t^2 + b_3 w_t + b_4 w_t^2 + e_t$
Model 3: $y_t = b_0 + b_1 t + b_2 t^2 + e_t$ Model 4: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + e_t$
Model 5: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + e_t$ Model 6: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + e_t$
Model 7: $y_t = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + b_6 t^6 + e_t$
Where y_t is milk yield at day in lactation t, t = days in lactation, $\gamma_t = t/305$, $w_t = \ln(305/t)$

* significant ($P < 0.05$) for Ho: LSMEAN=0, ** highly significant ($P < 0.01$) for Ho: LSMEAN=0.

Calving season subclasses. The pattern of significant results of the LS means of 305-d milk yield deviations (predicted minus actual 305-d milk yield) by breed group x lactation number x calving season subclasses was exactly the same as that of breed group x lactation number x calving age subclasses. Model 1 was the only one that had significant ($P < 0.01$) predicted minus actual 305-d milk yield differences, and they occurred in all subclasses. All the other models (2 to 7) produced nonsignificant differences. Again, model 3 would be the model of choice for its simplicity and economy of computations.

The overall ranking of the seven models for 305-d prediction of milk yield, considering the number of nonsignificant subclasses and their overall LS mean in both sets of subclasses, was as follows: 1) models 2 (mixed log second degree polynomial) and 3 (second degree polynomial), 2) model 5 (fourth degree polynomial), 3) model 4 (third degree polynomial), 4) model 7 (sixth degree polynomial), 5) model 6 (fifth degree polynomial), and 6) model 1 (gamma function).

Conclusions

Four distinct types of lactation curves were identified in the Holstein Friesian-Red Sindhi herd of SARTC based on the existence of a peak yield and the rate of descent of lactation curves. Types of lactation curves varied across breed group x lactation number x calving age and breed group x lactation number x calving season subclasses.

Models 1 through 7 had identical rankings when predicting daily milk yields and 305-d accumulated milk yields. The best ones were models 2 (mixed log second degree polynomial) and 3 (second degree polynomial), and the worst one was model 1 (gamma function). However, if computational requirements are also considered, then model 3 (second degree polynomial) would be the model of choice for this data set. For accumulated 100-d milk yield, the model with the smallest predicted minus actual milk yield differences was model 7 (sixth degree polynomial).

The ranking of these models should be taken with caution because of the small size of the data set used here. This study needs to be repeated with a larger Thai data set that has a more accurate representation of the cattle breeds and crossbred groups present in Thailand. In addition, the fact that the shape of lactation curves can vary by breed group, lactation, calving season, calving age, and possibly other factors, suggests that analyses like these need to be done periodically to determine the most appropriate equations to be used for genetic prediction purposes.

Accurate prediction of daily milk yields will help dairy producers to improve their feeding and management programs. In addition, accurate prediction of accumulated 100-d and 305-d milk yields will help improve the accuracy of genetic predictions of sires and dams. However, most dairy organizations in Thailand record daily milk yields on a monthly basis. Thus, the predictive ability of the seven models needs to be reevaluated using *monthly test-day records*, and their results compared to those obtained *here* using *all daily milk records*.

Acknowledgements

The authors are thankful for the financial support from the Thailand Research Fund under the Royal Golden Jubilee Project. The authors are grateful to the staff of the Sakon Nakhon Agricultural Research and Training Center for making their data set available for this research. The authors thank L. R. McDowell, T. A. Olson, and J. Rosales for reviewing the manuscript.

Literature Cited

Ali, T. E., and Schaeffer, L. R. 1987. Accounting for covariances among test day milk yields in dairy cows. Can. J. Dairy Sci. 67:637-644.

Batra, T. R. 1986. Comparison of two mathematical models in fitting lactation curves for pureline and crossline dairy cows. Can. J. Dairy Sci. 66:405-414.

Cobby, J. M. and Le Du, Y. L. P. 1978. On fitting curves to lactation data. Anim. Prod. 26:127-133.

Dodge, M. and Stinson, C. 1999. Running Microsoft Excel 2000. Microsoft Press, Washington, USA. 994p.

Grossman, M., Kuck, A. L., and Norton, H. W. 1986. Lactation curves of purebred and crossbred dairy cattle. J. Dairy Sci. 69:195-203.

Grossman, M., Hartz, S. M., and Koops, W. J. 1999. Persistency of lactation yield: a novel approach. J. Dairy Sci. 82:2192-2197.

Madalena, F. E., Martinez, M. L., and Freitas, A. F. 1979. Lactation curves of Holstein-Friesian and Holstein-Friesian x Gir cows. Anim. Prod. 29:101-107.

Morant, S. V. and Gnanasakthy, A. 1989. A new approach to the mathematical formulation of lactation curves. Anim. Prod. 49:151-162.

Rowlands, G. J., Lucey, S., and Russell, A. M. 1982. A comparison of different models of the lactation curve in dairy cattle. Anim Prod. 35:135-144

SARTC. 1999. Dairy and beef production under Northeast conditions. The 3rd Dairy - Beef workshop, February 8-12, 1999, Sakon Nakhon Agricultural Research and Training Center (SARTC), Rajamagala Institute of Technology, Sakon Nakhon, Thailand.

SAS. 1990. SAS/STAT User's Guide. 4th ed. SAS Institute Inc., Cary, NC.

Schaeffer, L. R., Minder, C. E., McMillan, I., and Burnside, E. B. 1977. Nonlinear techniques for predicting 305-day lactation production of Holsteins and Jerseys. J. Dairy Sci. 60:1636-1644.

Tekerli, M., Akinci, Z., Dogan, I., and Akcan, A. 2000. Factors affecting the shape or lactation curves of Holstein cows from the Balikesir province of Turkey. J. Dairy Sci. 83:1381-1386.

Vargas, B., Koops, W. J., Herrero, M., and Van Arendonk, J. A. M. 2000. Modeling extended lactation of dairy cows. J. Dairy Sci. 83:1371-1380.

Wood, P. D. P. 1967. Algebraic model of the lactation curve in cattle. Nature (Lond). 216:164-165.

Wood, P. D. P. 1972. A note on seasonal fluctuations in milk production. Anim. Prod. 15:89-92.